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NAVAL POSTGRADUATE SCHOOL

Monterey, California



NAVAL APPLICATIONS:
TEN ALGORITHMS FOR THE HEWLETT-PACKARD
HP-67 AND HP-97 CALCULATORS

edited by

R. H. Shudde

February 1979

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NAVAL POSTGRADUATE SCHOOL
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<p>Ten algorithms pertaining to underwater acoustics target motion analysis, P-3 mission planning, flight crew management, and naval gunfire support conversion are presented along with programs for Hewlett-Packard HP-67 and HP-97 programmable calculators.</p>		

NAVAL APPLICATIONS:
TEN ALGORITHMS FOR THE HEWLETT-PACKARD
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The programs in this report are for use within the Department of the Navy, and they are presented without representation or warranty of any kind.

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ABSTRACT

Ten algorithms pertaining to underwater acoustics, target motion analysis, P-3 mission planning, flight crew management, and naval gunfire support conversions are presented along with programs for Hewlett-Packard HP-67 and HP-97 programmable calculators.

I. INTRODUCTION

This report contains a collection of programs which were submitted by officers in partial fulfillment of the requirements of the course Tactical Design and Analysis (OA 4658) conducted at the Naval Postgraduate School during the period of October through December 1978.

All programs were listed using an HP-97 with HP-97 key codes. The corresponding HP-67 key codes may be found on pages 324 through 331 of the "HP-67 Owner's Handbook and Programming Guide."

II. ACTIVE SONAR ACQUISITION by Mr. R. F. Fish and LT M. H. Trent

A. Problem Statement

A sonar at a depth (SD) has the possibility of detecting a target at a depth TD at a slant range r' . Detection can only occur if the target lies within the beam pattern and the signal excess is at least equal to the detection threshold. Whether or not the system is noise or reverberation limited depends on the geometry and doppler frequency shift. The problem is to determine the acquisition range of the sonar with various geometries and acoustic parameters.

B. Operational Analysis

The analysis uses a 0 dB detection threshold because of the limited number of storage registers (26) and program steps (224) in the calculator.

In using the program it should be noted that calculations do not include the effect of shadow zones. Acquisition ranges computed must be considered with this in mind. In addition, once the target and surface signals are outside the beam pattern ($\pm 3\text{db}$) they are assumed to abruptly disappear respectively. The analysis also assumes the water is deep with no bottom effects.

Considering the above caveats the source and target can be placed as desired in the medium and the appropriate sonar equation parameters will be computed. In doing so, several tests will be made to determine which equations will be used.

The program will terminate before acquisition if one of the following occurs:

Slant range

$$r' < 0,$$

Angle-to-target

$$\theta_1 > \phi/2 + \gamma, \text{ or } \theta_1 > \phi/2 - \gamma \quad (\text{if target is below source}),$$

Angle-to-surface at r'

$$\theta_2 > \phi/2 + \gamma,$$

where

γ = pitch angle range.

These terminations and signal excess $SE \geq 0$ will finish with a "1" printed as an output at the end of the calculations.

After calculation of the surface reverberation level, RL_s , there is a program stop where the appropriate correction can be input for off-axis transmission and reception. The same event occurs when SE is calculated so that the sonar equation can be corrected.

The output listing is as follows:

<u>Doppler</u>	<u>No Doppler</u>
r'	r'
θ_1	θ_1
θ_2	θ_2
TL	TL
NL_s	RL_s displays but not printed
SE	Total of $RL_s + RL_v + NL_s$ combined and SE

C. Computational Algorithm

1. Input

--Sound speed, c (m/sec)
--Listening time between transmit pulses, t (sec)
--Source depth, SD (m)
--Target depth, TD (m)
--Horizontal (and vertical) half-beam width, $\phi/2$ (degrees)
--Sonar pitch angle, γ (degrees)
--Mixed layer depth, D (meters)
--Absorption coefficient, α (dB/meter)
--Frequency, f (kHz)
--Sea state, S.S.
--Column scattering coefficient, S_c
--Constant, 10
--Sonar self-noise level, NL_s (dB)
--Target strength, TS (dB)
--Range decrement (meters)
--[Pulse length, τ (sec) $\times \phi$ (radians)] $\div 2$
--Surface scattering coefficient, S_s (dB)
--Sonar source level, SL (dB)

2. For doppler set Flag 0; for no doppler clear Flag 0.

3. Output

r' , θ_1 , θ_2 , TL , RL_s , RL_v . total level, SE. For doppler, RL_s , RL_v , and total level are included in NL_s .

D. HP-67/97 Calculator Program

1. User Instructions

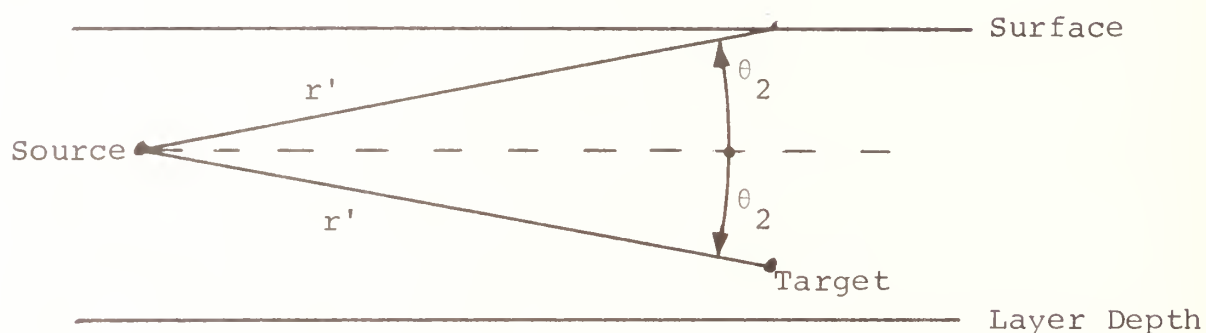
Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Enter data in primary stage:			
a.	Sound speed (m/sec)	c	STO 0	
b.	Listening time (sec)	t	STO 1	
c.	Source depth (m)	SD	STO 3	
d.	Target depth (m)	TD	STO 4	
e.	Half-beam width (deg)	$\phi/2$	STO 7	
f.	Sonar pitch angle (deg)	γ	STO 8	
g.	Mixed layer depth (m)	D	STO A	
h.	Absorption coefficient (dB/m)	α	STO B	
i.	Frequency (kHz)	f	STO C	
j.	Sea state	S.S.	STO D	
k.	Source level (db)	SL	h STI	
3.	Enter data in secondary storage:*		P ∇ S	
a.	Column scattering coefficient	S_C	STO 0	
b.	Constant	10	STO 2	
c.	Sonar self-noise level	NL _S	STO 3	
d.	Target strength	TS	STO 4	
e.	Range decrement (m)	r.d.	STO 5	
f.	$[\tau(\text{sec}) \times \phi(\text{radians})]/2$	$\tau\phi/2$	STO 6	
g.	Surface scattering coefficient	S_S	STO 8	
4.	Primary/Secondary exchange*		P ∇ S	
5.	Doppler or No doppler		SF 0 CF 0	doppler no doppler
6.	Start computations	--	A	See Step 7
7.	Printed output:			
a.	Slant range			r'
b.	Angle-to-target			θ_1
c.	Angle-to-surface at range r'			θ_2
d.	Transmission Loss			TL
e.	If Flag 2 is set, self-noise is printed.			NL _S

Step	Instruction	Input	Key(s)	Output
8.	If Flag 2 is set, go to Step 9. Otherwise:			
a.	Display surface reverberation level			RL_s
b.	Enter two-way beam pattern correction in db (0 if no correction) Print total corrected level.	Correction	R/S	Total RL_s
9.	Stop and display SE. To change range decrement, execute Step a. Otherwise go to Step b.			SE
a.	Key in new range decrement	r.d.	STO 5 R ↓	SE
b.	Enter two-way beam pattern correction in db (0 if no correction) Display corrected SE	Correction	R/S	Corrected SE
10a.	If $SE < 0$, execution continues from Step 7.			
b.	If $SE \geq 0$, termination occurs			1.00

* The primary and secondary registers must be exchanged before (Step 3) and after (Step 4) entering data into the secondary storage registers.

2. Sample Problems

- a. Sonar and target are in the mixed layer in a "doppler" situation, so that acquisition is noise limited by NL_s (Figure 1).



Source depth = 30 meters

Target depth = 60 meters

Mixed Layer depth = 75 meters

FIGURE 1. Geometry of Sample Problem 1.

Input.

R0: 1500 m/sec	S0: -50 dB
R1: 6.7 sec	S2: 10
R3: 30m	S3: 65 dB re 1 μ Pa
R4: 60m	S4: 10 dB
R7: 10°	S5: 500 m (initial range decrement)
R8: 5°	S6: .17°sec
RA: 75m	S8: -30 dB
RB: .00328 dB/m	
RC: 25 kHz	
RD: 2	
RI: 227 dB re 1 μ Pa @ 1 m	

Set Flag 0. No corrections are made to SE.

Output

The results are shown on the Sample Problem output.

At acquisition

$r' = 3650$ meters
 $\theta_1 = .47$
 $\theta_2 = .47$
TL = 85.9 dB (layer)
 $NL_s = 65$ dB re 1 μ Pa
SE = .21 dB

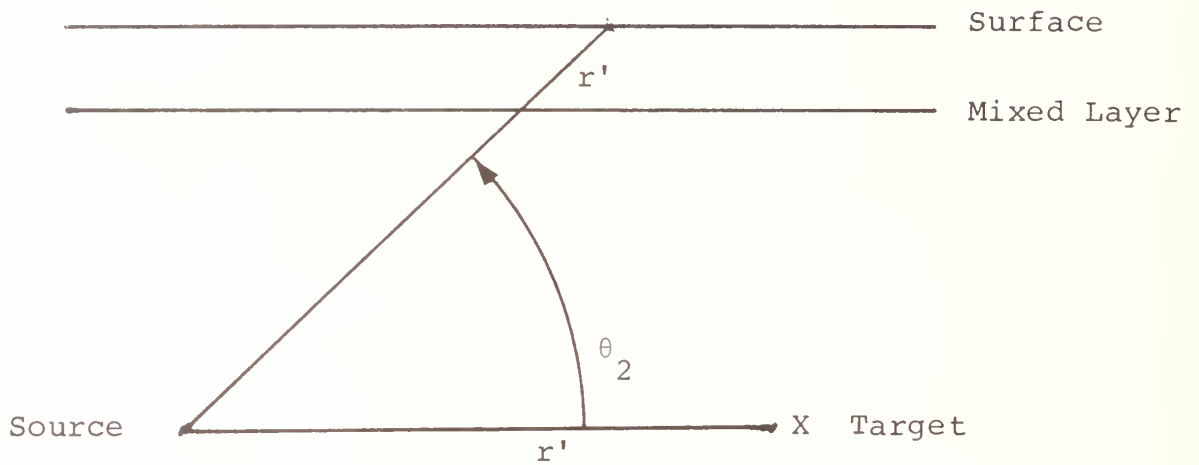
1500.00 ST00	}	Primary Storage Registers
6.70 ST01		
33.00 ST02		
50.00 ST03		
10.00 ST04		
5.00 ST05		
70.00 ST06		
.00000 ST07		
25.00 ST08		
2.00 ST09		
127.00 ST10		
F20		
-50.00 ST00	}	Secondary Storage Registers
10.00 ST01		
65.00 ST02		
10.00 ST03		
500.00 ST04		
.17 ST05		
-30.00 ST06		
F20		

SAMPLE PROBLEM 1. Input Data

SFO	Set for doppler
GSEA	Start
5025.00 ***	r'
0.34 ***	θ_1
0.34 ***	θ_2
93.76 ***	TL
65.00 ***	RL _s
0.00 R E	Correction to SE
-15.52 ***	Corrected SE
4525.00 ***	
0.36 ***	
0.38 ***	
90.95 ***	
65.00 ***	
0.00 R E	
-9.90 ***	
4025.00 ***	
0.43 ***	
0.43 ***	
88.03 ***	
65.00 ***	
300.00 ST05]	Change range decrement
R E	to 300 meters
0.00 R E	
-4.17 ***	
3725.00 ***	
0.46 ***	
0.46 ***	
86.34 ***	
65.00 ***	
50.00 ST05]	Change range decrement
R E	
0.00 R E	
-0.67 ***	
3675.00 ***	
0.47 ***	
0.47 ***	
86.04 ***	
65.00 ***	
25.00 ST05]	Change range decrement
R E	
0.00 R E	
-0.05 ***	
3650.00 ***	r'
0.47 ***	θ_1
0.47 ***	θ_2
85.90 ***	TL (layer)
65.00 ***	NL _s
0.00 R E	
0.21 ***	SE = 0.21
1.00	Terminate

SAMPLE PROBLEM 2. Output

b. Sample Problems 2 and 3. The sonar and target configuration are shown in Figure 2.



Source depth = 600 meters
Target depth = 600 meters
Mixed Layer depth = 75 meters

FIGURE 2. Geometry of Sample Problems 2 and 3.

Input for Problems 2 and 3:

Same as for Problem 1 except

R3: 600 meters Flag 0: set for problem 2;
R4: 600 meters clear for problem 3
R8: 0°

0 entered as correction factors to RL_s and SE.

Output for Problem 2:

$r' = 4125$ meters

$\theta_1 = 0^\circ$

$\theta_2 = 8.36^\circ$

SE = .32 dB

Sonar acquired the target at about 4125 meters.

Output for Problem 3:

$r' = 3025$ meters

$\theta_1 = 0^\circ$

$\theta_2 = 11.44^\circ$

SE = 0.85 dB

Sonar acquired at 3025 meters.

	SFO	
	GSBA	
5025.00	***	
0.00	***	
6.86	***	
90.50	***	
25.00	***	
0.00	R/S	
-9.01	***	
4525.00	***	
0.00	***	
7.62	***	
87.55	***	
65.00	***	
100.00	STOE	Change range decrement to 100 meters
	R↓	
0.00	R/S	
-3.91	***	
4425.00	***	
0.00	***	
7.79	***	
87.43	***	
65.00	***	
0.00	R/S	
-2.86	***	
4325.00	***	
0.00	***	
7.97	***	
86.91	***	
65.00	***	
0.00	R/S	
-1.81	***	
4225.00	***	
0.00	***	
8.16	***	
86.37	***	
65.00	***	
0.00	R/S	
-0.75	***	
4125.00	***	r'
0.00	***	θ_1
8.36	***	θ_2
85.84	***	TL
65.00	***	NL _s
0.00	R/S	
0.32	***	SE
1.00		Terminate

SAMPLE PROBLEM 2. Output

	CFD	No doppler
	GSEA	Start
5025.00	***	r'
0.00	***	θ_1
6.86	***	θ_2
90.50	***	TL
77.07	***	RLS
0.00	R/S	No correction to RL_s
77.37	***	Total RL_s
0.00	R/S	No correction to SE
-21.38	***	Corrected SE
4525.00	***	
0.00	***	
7.62	***	
87.95	***	
81.71	***	
0.00	R/S	
81.85	***	
0.00	R/S	
-20.76	***	
4025.00	***	
0.00	***	
8.57	***	
85.30	***	
86.52	***	
0.00	R/S	
86.59	***	
0.00	R/S	
-20.19	***	
3525.00	***	
0.00	***	
9.80	***	
82.51	***	
91.53	***	
0.00	R/S	
91.58	***	
0.00	R/S	
-19.59	***	
3025.00	***	r'
0.00	***	θ_1
11.44	***	θ_2
79.54	***	
77.08	***	
0.85	***	
0.00	R/S	
0.85	***	SE
1.00		Termination

SAMPLE PROBLEM 3. Output

3. Program Storage Allocations and Program Listings

Registers

R0: c (m/sec)	S0: S_c (dB)
R1: t (sec) and $SL - 2TL + 10 \log r'$	S1: RL_v (dB)
R2: R_{max} and r'	S2: 10
R3: SD (m)	S3: NL (dB _{relμPa})
R4: TD (m)	S4: TS (dB)
R5: θ_1 (deg)	S5: Range decrement (m) (500 m default)
R6: θ_2 (deg)	S6: τ (sec) $\times \phi/2$ (AD)
R7: $\phi/2$ (deg)	S7: $10 \log(\phi\tau c/2)$
R8: γ (deg)	S8: S_s (dB)
R9: $\phi/2 + \gamma$ (deg)	S9: RL_s (dB)
RA: D (meters)	
RB: α (dB/meter)	
RC: f (kHz)	
RD: S.S.	
RE: TL (dB)	
RI: SL (dB _{relμPa} @ 1m) or consistent with NL	

Initial Flag Status and Use:

0: ON for doppler, OFF for no doppler	1, 2, 3: OFF, unused
--	----------------------

User control keys:

A:	Start program	a:
B:		b:
C:		c:
D:		d:
E:		e:

001	*LBLH	21 11	038	X>Y?	16-34	075	X>Y?	16-34
002	RCL0	36 00	039	GT00	22 00	076	GT06	22 00
003	FCL1	36 01	040	*LBL1	21 01	077	L06	16 33
004	-	-35	041	RCL7	36 07	078	1	01
005	2	02	042	RCL8	36 08	079	0	00
006	=	-34	043	-	-45	080	*	-35
007	ST02	35 02	044	RCL5	36 05	081	RCL2	36 02
008	*LELB	21 12	045	X>Y?	16-34	082	L06	16 33
009	RCL2	36 02	046	GT00	22 00	083	1	01
010	X<0?	16-45	047	RCLH	36 11	084	0	00
011	GT00	22 00	048	RCL3	36 03	085	*	-35
012	FRTX	-14	049	X>Y?	16-34	086	+	-55
013	1 2	55	050	GT06	22 06	087	RCLF	36 12
014	RCL3	36 03	051	X>Y	-41	088	RCL2	36 02
015	RCL4	36 04	052	RCL4	36 04	089	X	-35
016	-	-45	053	X>Y?	16-34	090	+	-55
017	HES	16 31	054	GT06	22 06	091	RCLC	36 12
018	-	-35	055	RCL3	36 03	092	*W	54
019	SIN4	16 41	056	RCL4	36 04	093	RCLD	36 14
020	ST05	35 05	057	X<Y?	16-35	094	X	-35
021	FRTX	-14	058	X>Y	-41	095	1	01
022	RCL3	36 03	059	CHS	-55	096	*	-55
023	FCL2	36 02	060	RCLH	36 11	097	0	00
024	=	-34	061	+	-55	098	4	04
025	SIN4	16 41	062	X=0?	16-43	099	X	-35
026	ST06	35 06	063	GT06	22 06	100	RCL2	36 02
027	FRTX	-14	064	1/X	52	101	X	-35
028	RCL7	36 07	065	RCLH	36 11	102	RCLH	36 11
029	FCL3	36 03	066	*2	53	103	*X	54
030	+	-55	067	X	-35	104	8	00
031	ST09	35 09	068	*X	54	105	4	04
032	RCL4	36 04	069	1	01	106	0	00
033	RCL3	36 03	070	0	02	107	X	-35
034	X<Y?	16-35	071	5	05	108	=	-34
035	GT01	22 01	072	X	-35	109	+	-55
036	RCL9	36 09	073	RCL2	36 02	110	STOE	35 15
037	RCL5	36 05	074	X>Y	-41	111	GT02	22 12

112	*LBL6	21 06	149	GT06	22 15	187	+	-55
113	RCL2	36 02	150	RCL9	36 02	188	P+S	16-51
114	LOG	16 32	151	RCL6	36 06	189	*LBLE	21 15
115	2	02	152	X>Y?	16-34	190	F+S	16-51
116	0	02	153	GT00	22 14	191	RCL3	36 03
117	X	-35	154	RCL1	36 01	192	RCL2	36 02
118	RCL5	36 11	155	F+S	16-51	193	÷	-24
119	RCL2	36 02	156	RCL7	36 2	194	RCL2	36 02
120	X	-35	157	+	-55	195	X+Y	-41
121	+	-55	158	RCL8	36 06	196	YX	31
122	STOE	35 15	159	+	-55	197	+	-55
123	*LBLE	21 15	160	ST09	35 03	198	LOG	16 32
124	FRTX	-14	161	F+S	51	199	RCL2	36 02
125	RCL1	36 46	162	-	-45	200	=	-35
126	RCL2	36 15	163	GT0A	22 16 11	201	FRTX	-14
127	2	02	164	*LELD	21 14	202	CHS	-22
128	X	-35	165	P+S	16-51	203	RCL1	36 46
129	-	-45	166	0	02	204	+	-55
130	RCL2	36 02	167	ST09	35 03	205	RCL2	36 15
131	LOG	16 32	168	*LBLE	21 16 11	206	2	02
132	1	01	169	RCL0	36 00	207	X	-35
133	0	02	170	RCL7	36 07	208	-	-45
134	X	-35	171	+	-55	209	RCL4	36 04
135	+	-55	172	P+S	16-51	210	+	-55
136	ST01	35 01	173	RCL1	36 01	211	R+S	51
137	RCL6	36 00	174	+	-55	212	-	-45
138	P+S	16-51	175	P+S	16-51	213	FRTX	-14
139	RCL6	36 02	176	ST01	35 01	214	X+00	16-45
140	X	-35	177	RCL2	36 02	215	6109	22 03
141	LOG	16 32	178	RCL9	36 03	216	*LBLE	21 03
142	1	01	179	RCL2	36 01	217	1	01
143	0	02	180	÷	-24	218	FRTX	-14
144	ST02	35 01	181	YX	31	219	RTN	24
145	X	-35	182	RCL2	36 02	220	*LBLE	21 03
146	ST07	35 07	183	RCL1	36 01	221	RCL5	36 03
147	P+S	16-51	184	RCL2	36 02	222	F+S	16-51
148	F00	16 23 02	185	÷	-24	223	ST-2	35-45 02
			186	YX	31	224	GT06	22 12

E. Computational Analysis

The active sonar equation is

$$SL - 2TL + TS - (NL - DI) \geq DT ,$$

NL_s
 RL

where

SL = source level for the sonar (dB_{relμPa} @ 1m),

TL = transmission loss (dB),

(NL-DI) = ambient noise term which is neglected due to
a higher NL_s or RL ,

RL = reverberation level (dB)

DT = system detection threshold (0dB assumed),

TS = target strength (dB),

and NL_s = Self-noise (dB_{relμPa}).

The only terms not known in the equation, TL and RL_s , are calculated at various ranges (decrements) until the signal excess (SE) $\geq DT(0)$.

The TL is calculated for two conditions:

a. When both source and target are in the layer (Reference 1)

$$TL = 10 \log r_t + 10 \log r' + \alpha r' + \frac{br'}{rs} ,$$

where

$r_t = 105 \sqrt{\frac{D^2}{D - z_s}}$ is the transition range (meters),

α = absorption (dB/meters),

$b = 1.04 \times SS \times \sqrt{f}$ bounce factor (dB/bounce) valid
between (3-25 kHz) (3-14 dB/bounce),

$r_s = 840 \sqrt{D}$,

z_s = larger of source or receiver depths (meters),

and D = layer thickness (meters).

b. When both source and target are not in the layer the TL is

$$TL = 20 \log r' + \alpha r' \quad (\text{for } r' < r_t \text{ also when in layer}).$$

After using the proper TL formula it must be decided whether or not there is sufficient doppler to be able to disregard reverberation (i.e., RL_s and RL_v). If there is enough doppler then the NL_s term dominates and the sonar equation can be solved. If there is no doppler the appropriate reverberation must be considered and combined with NL_s . Then the sonar equation can be solved. By successive decrements of r' , there may be a point where $SE \geq DT$ and thus detection has occurred.

The reverberation equations are (Reference 2),

$$RL_s = SL - 2TL + 10 \log r' + S_s + 10 \log \left(\frac{\phi C \tau}{2} \right) ,$$

where

S_s = surface scattering parameter (dB) for the particular conditions (wind speed, grazing angle),

ϕ = sonar horizontal beam width (radians),

c = wave propagation time

and τ = transmit pulse width (seconds);

and

$$RL_v = SL - 2TL + 10 \log r' + S_c + 10 \log \left(\frac{\phi C \tau}{2} \right),$$

where

S_c = column scattering coefficient (dB) for the particular environmental conditions.

At each range decrement θ_1 and θ_2 are calculated to determine if they are inside the beam pattern. If θ_1 is not, acquisition cannot occur. If θ_2 is not, then RL_s is not important. The formulae for these quantities are (Figure 3):

$$\theta_1 = \sin^{-1} \left| \frac{SD - TD}{r'} \right| \quad \text{and} \quad \theta_2 = \sin^{-1} \frac{SD}{r'},$$

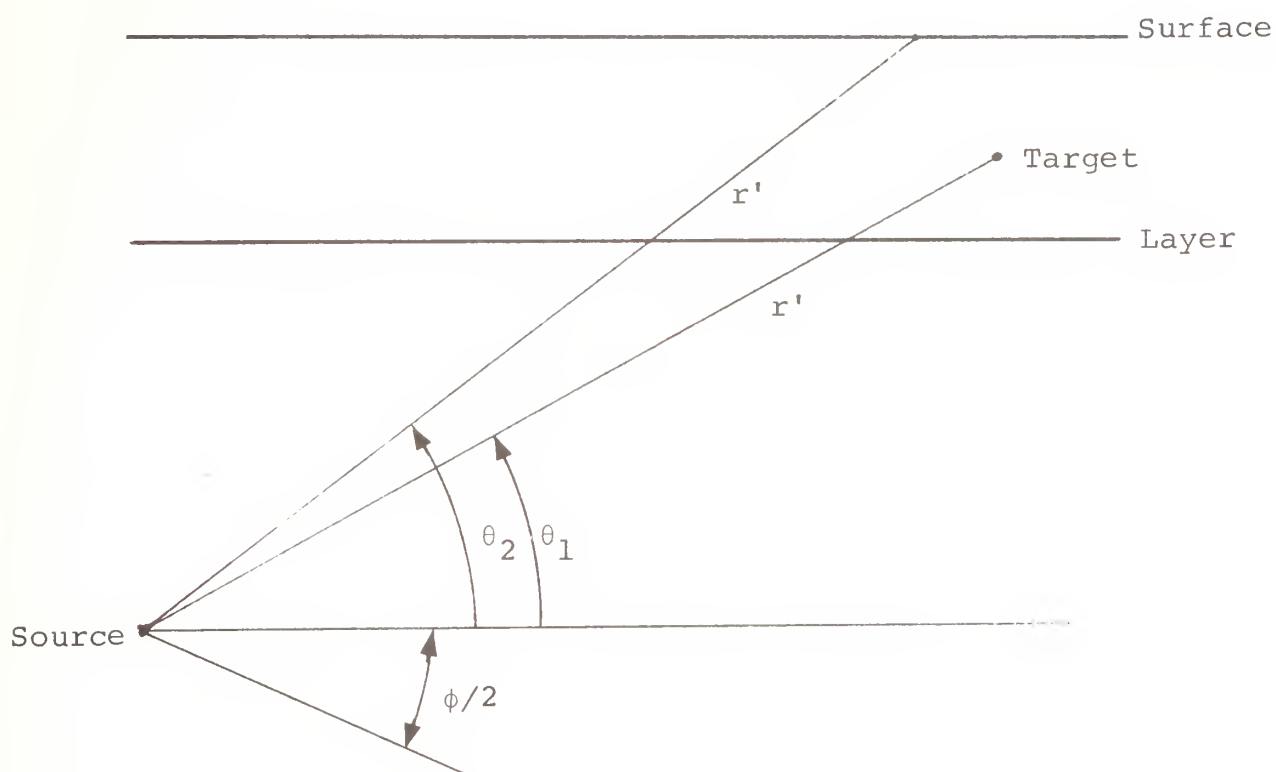
where

SD = source depth,

and TD = target depth.

In addition, depending on the values of these angles, corrections can be made to the RL_s and sonar equation to compensate for off-axis (beam pattern) transmission and reception.

To get an initial value of $r' = R_{\max}$ the equation $R_{\max} = ct/2$ is used where t = the sonar "listening" time or time between successive pulse transmission.



- ϕ = sonar beam pattern (3dB points)
- θ_1 = angle to target at range r'
- θ_2 = angle to surface at range r'
- r' = slant range
- D = layer depth

FIGURE 3. Source and Target Geometry

F. References

1. A. B. Coppens and J. V. Sanders, "An Introduction to the Sonar Equations with Applications," Technical Report NPS-61Sd76071, July 1976, Naval Postgraduate School, Monterey, CA 93940.
2. R. J. Urick, Principles of Underwater Sound, 2nd Edition McGraw-Hill Book Co., 1975.

III. THREE ENGINE AVAILABLE RANGE REMAINING: P-3(B) AIRCRAFT
CONFIGURATION "B" by LT R. J. Knight

A. Problem Statement

Aircraft total fuel remaining, outside air temperature, and aircraft altitude data are available. Determine the aircraft's available range remaining.

This program allows a pilot or copilot to rapidly and efficiently provide a quick estimate of available range remaining in an emergency situation (three engine flight).

B. Operational Analysis

The aircraft's available range remaining can be extracted from the table listed on pages 12-189 of the P-3(B) aircraft NATOPS manual.

C. Computational Algorithm

1. Input fuel remaining (pounds).
2. Input outside temperature ($^{\circ}\text{C}$).
3. Input altitude (feet)
4. Calculate the three-engine available range remaining.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Load program magnetic card side #1 and side #2			0.000000000
2.	Input fuel remaining,* press enter	pounds	ENT	pounds
3.	Input outside air temperature for specific altitude press enter	°C	ENT	°C
4.	Input altitude	feet		feet
5.	Press A to calculate the three engine available range remaining		A	range in NM

* Note: Fuel remaining must be entered as #10,000, #20,000, #30,000, #40,000 or #50,000. "Error" will display otherwise.

2. Sample Problem

Calculate the three-engine available range remaining if the remaining fuel is 10,000 pounds, the outside temperature is 11°C, and the altitude is 2000 feet. (ANSWER: 295 n.mi.)

```
10000. ENT1
  11. ENT1
 2000. GSEn
 295.  ***
```

3. Program Storage Allocation, Permanent Data, and Program Listing

Registers:

R0:	a ₀	S0:	A: Fuel
R1:	b ₀	S1:	B: Altitude
R2:	a ₁	S2:	C: Temperature
R3:	b ₁	S3:	D: Temperature deviation
R4:	a ₂	S4:	E: Uncorrected range.
R5:	b ₂	S5:	
R6:	a ₃	S6:	
R7:	b ₃	S7:	
R8:	a ₄	S8:	
R9:	b ₄	S9:	

Flags: OFF, unused.

User Control Keys:

A: Compute	a.
B:	b:
C:	c:
D:	d:
E:	e:

Permanent Data

The following permanent data are stored in the primary storage registers R0 through R9.

-34382.26904	0
122.9065370	1
-35846.62839	2
42.18615709	3
-56857.62093	4
27.00641670	5
-38068.60581	6
20.18918143	7
-39482.60271	8
16.50293201	9

Program Listing

```

001 *LECH 21 11
002 STOB 35 12
003 FJ -31
004 STOB 35 12
005 FJ -31
006 STOB 35 11
007 EEX -23
008 4 04
009 ÷ -24
010 1 01
011 - -45
012 X=0? 16-43
013 GT01 22 01
014 1 01
015 - -45
016 X=0? 16-43
017 GT02 22 02
018 1 01
019 - -45
020 X=0? 16-43
021 GT03 22 03
022 1 01
023 - -45
024 X=0? 16-43
025 GT04 22 04
026 1 01
027 - -45
028 X=0? 16-43
029 GT05 22 05
030 XZY -41
031 0 00
032 ÷ -24
033 *LBL1 21 01
034 RCLB 36 12
035 RCL0 36 00
036 - -45
037 RCL1 36 01
038 ÷ -24
039 STOE 35 15
040 GT0e 22 16 15
041 *LBL2 21 02
042 RCLB 36 12
043 RCL2 36 02
044 - -45
045 RCL3 36 03
046 ÷ -24
047 STOE 35 15
048 GT0e 22 16 15
049 *LBL3 21 03
050 RCLB 36 12
051 RCL4 36 04
052 - -45
053 RCL5 36 05
054 ÷ -24
055 STOE 35 15
056 GT0e 22 16 15
057 *LBL4 21 04

```

```

START
STO ALTITUDE

STO TEMP

STO FUEL

TEST FUEL
= 10,000 lbs?

```

```

TEST FUEL
= 20,000 lbs?

```

```

TEST FUEL
= 30,000 lbs?

```

```

TEST FUEL
= 40,000 lbs

```

```

TEST FUEL
= 50,000 lbs

```

```

TEST FUEL
> 50,000 lbs

```

```

COMPUTE RANGE
W/O TEMP CORRECTION
F = 10,000 lbs

```

```

COMPUTE RANGE
W/O TEMP CORRECTION
F = 20,000 lbs

```

```

COMPUTE RANGE
W/O TEMP CORRECTION
F = 30,000 lbs

```

```

058 RCL4 36 04
059 RCL5 36 12
060 RCL6 36 06
061 - -45
062 RCL7 36 07
063 ÷ -24
064 STOE 35 15
065 GT0e 22 16 15
066 *LBL5 21 05
067 RCLB 36 12
068 RCL8 36 08
069 - -45
070 RCL9 36 09
071 ÷ -24
072 STOE 35 15
073 *LBL6 21 16 15
074 RCLB 36 12
075 EEX -23
076 3 03
077 ÷ -24
078 2 02
079 CHS -22
080 X -35
081 1 01
082 5 05
083 + -55
084 RCLC 36 13
085 - -45
086 STOD 35 14
087 X=0? 16-43
088 GT0W 22 16 14
089 RCLC 36 15
090 DSP0 -63 00
091 RTN 24
092 *LBLd 21 16 14
093 . -62
094 0 00
095 0 00
096 2 02
097 X -35
098 RCLC 36 15
099 X -35
100 CHS -22
101 RCLC 36 15
102 + -55
103 DSP0 -63 00
104 R/S 51

```

```

COMPUTE RANGE
W/O TEMP CORRECTION
F = 40,000 lbs

```

```

COMPUTE RANGE
W/O TEMP CORRECTION
F = 50,000 lbs

```

```

COMPUTE TEMP
CORRECTION

```

```

IF TEMP = STD
DISPLAY RANGE

```

```

TEMP ≠ STD
DISPLAY RANGE

```

```

END

```

E. Mathematical Analysis

A linear curve fit was performed using the HP-67/97 standard pack SD-03A program. Five "fits" were performed. For a constant fuel weight, X represented the range and Y represented the altitude. Resulting outputs provided the following:

<u>Fuel (pounds)</u>	<u>R²</u>	<u>a</u>	<u>b</u>
10,000	.998615613	-34,302.26904	122.9065370
20,000	.998739204	-35,846.62839	43.18615709
30,000	.999756772	-36,857.62093	27.00641670
40,000	.999823326	-38,068.60561	20.18918143
50,000	.999802631	-39,482.60271	16.50293201

For a constant fuel the range X could be obtained as follows:

$$X = \frac{Y - a}{b}$$

or

$$\text{Range} = \frac{\text{altitude} - \text{intercept (constant fuel)}}{\text{slope (constant fuel)}}$$

Temperature correction:

increase range 1% per 5°C above standard

decrease range 1% per 5°C below standard .

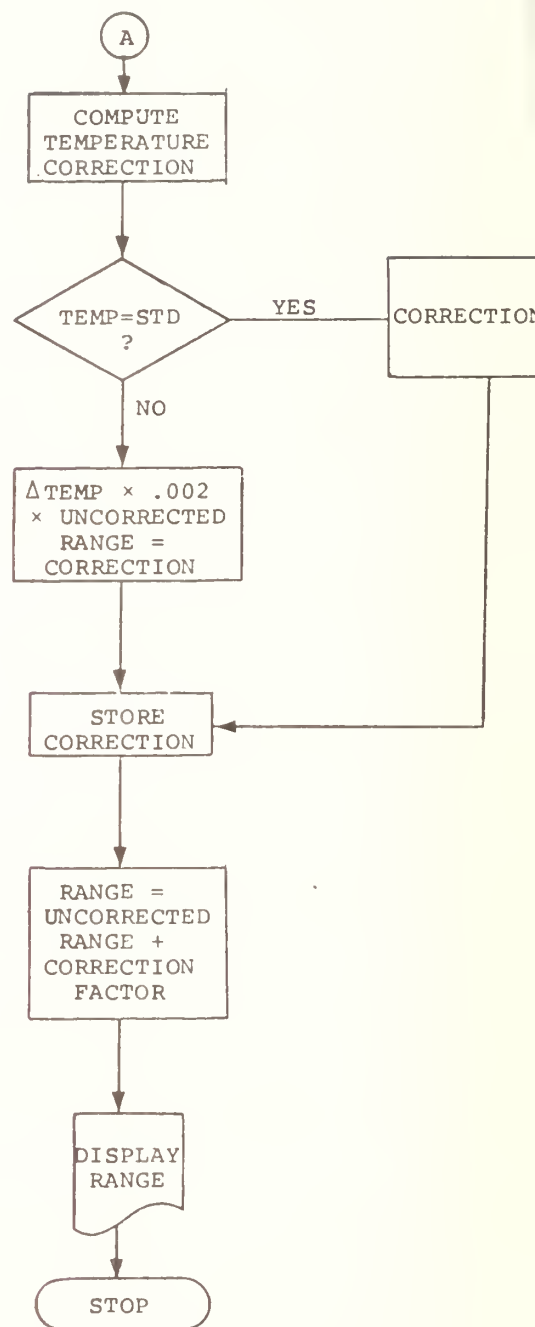
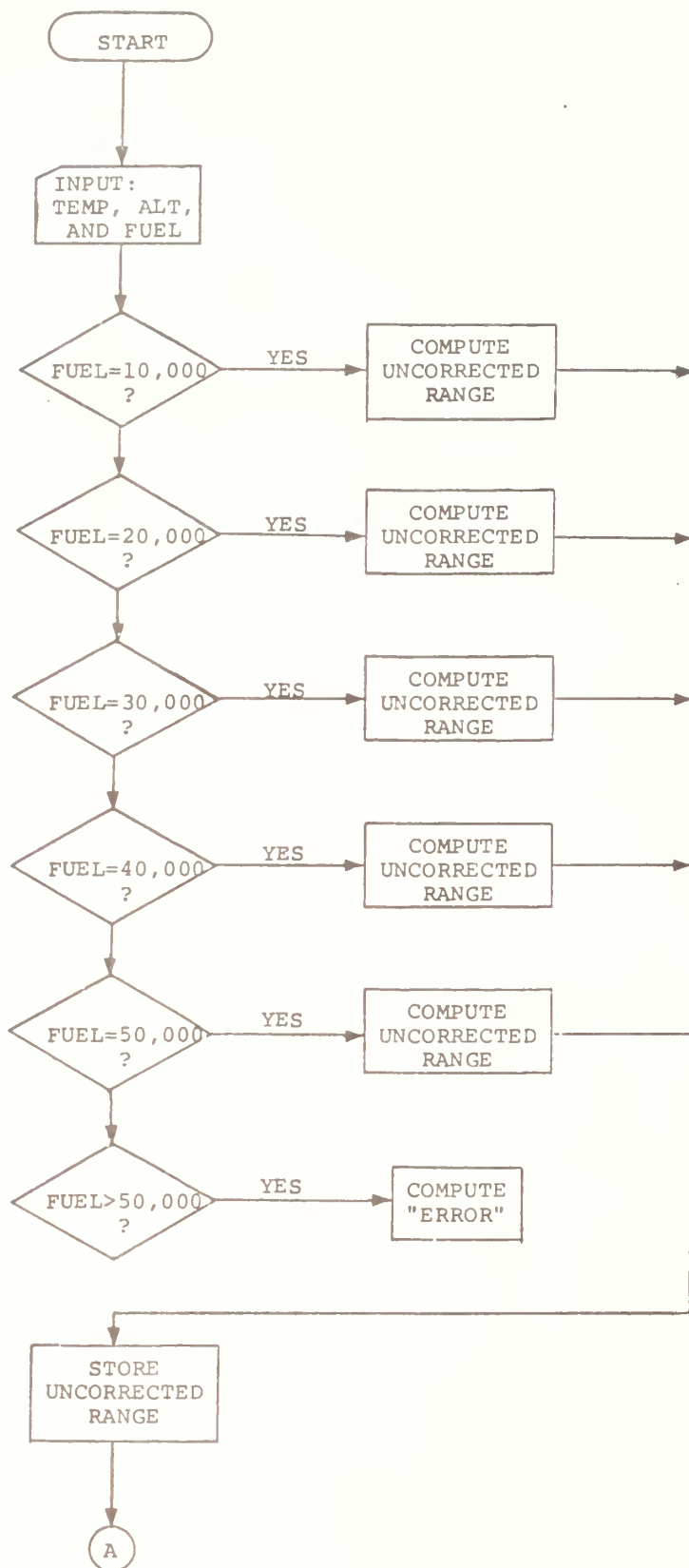
Based on a lapse rate of $-2^{\circ}/1000\text{ft}$ the standard temperature for a specified altitude is obtained by:

$$-2 \times \frac{\text{altitude}}{1000} + 15 = \text{standard temperature (for altitude)}$$

Subtracting the input temperature from the computed standard temperature yielded the difference in $^{\circ}\text{C}$ from standard.

A correction factor could be obtained for each difference times $.002/\text{degree}$ times uncorrected range. The range could then be computed by summing the correction factor and the uncorrected range value.

F. Program Flowchart



IV. MISSION PLANNING: P-3(B) AIRCRAFT CONFIGURATION "B"

LT M. D. Thomas.

A. Problem Statement

In order to carry out the various operational missions assigned the P3-B Aircraft, effective utilization of the platform is essential. All aspects of the mission must be carefully planned. Fuel planning directly influences endurance and the effectiveness of the mission. The NATOPS manual provides charts for this purpose. Two vital charts for planning are:

1. four engine maximum range operating table; used in proceeding to the operational area.
2. three engine loiter operating table; used while onstation for minimum fuel consumption.

The pilot or flight engineer enters with the aircraft's altitude and gross weight and finds the correct indicated airspeed (IAS) to fly.

This program is a user's program in that it translates these two charts onto an HP-67/97 magnetic card and allows calculation of IAS without the charts. Most missions are flown in configuration 'B' therefore the program presented here is for that case.

B. Operational Analysis

None.

C. Computational Algorithm

1. Enter altitude and gross weight in packed form: AAAAA.WWWW where AAAAA denotes the altitude in feet, and WWWW is the gross weight divided by 100,000. The leading zeroes, if any, in the value of WWW must be entered. For example, 18,000 feet and 76,500 pounds are entered as AAAAA = 18,000 and WWWW = 0765, that is 18,000.0765.
2. Compute the maximum range IAS or the three-engine loiter IAS.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Enter altitude and gross weight. G.W. must be at least a three digit number. Compute four engine max range IAS.	ALT.GW	A	IAS (max range)
3a.	Enter altitude and gross weight. G.W. must be at least a three digit number. Compute three engine loiter range IAS	ALT.GW	B	IAS (Loiter)
b.	Optional: Compute the four engine max range IAS without re-entering the altitude and weight		R/S	IAS (max temp)

2. Sample Problems

		A <u>Max Range IAS</u>	B <u>Loiter IAS</u>
1.	130,000 lbs at 18,000 ft 18000.130	252	220
2.	86,000 lbs at 6,000 ft 6000.086*	241	175
3.	76,500 lbs at 10,000 ft 10000.0765	232	165
4.	50,000 lbs at 3,000 ft 3000.050	Error	Error
5.	130,000 lbs at 30,000 ft 30000.130	Error	Error

*
Gross weight must be at least a three digit number (IE)

130,000	.130
76,000	.076
82,500	.0825

3. Program Storage Allocation and Listing

Registers

R0: altitude	S0:	RA:
R1: max range constant	S1:	:
R2: loiter airspeeds	S2:	:
:	:	RE:
R9:	S9:	RI:

Initial Flag Status and Use:

0: OFF, unused	2: OFF, unused
1. OFF, unused	3: OFF, unused.

User Control Keys:

A: Compute four engine IAS	a:
B: Compute three engine IAS	b:
C:	c:
D:	d:
E:	e:

001	*LEL4	21 11	UNPACK ALTITUDE AND GROSS WEIGHT	038	5	05	COMPUTE MAX RANGE IAS
002	DEF8	-63 06		039	XZ99	16-35	
003	ENT1	-21		040	6566	23 06	
004	INT	16 34		041	ROL0	36 06	
005	ST00	35 06		042	ROL1	36 01	
006	R4	-31		043	EEX	-23	
007	FRC	16 44		044	X	05	
008	EEX	-23		045	X	-35	
009	3	02		046	-	-45	
010	X	-35		047	EEX	-23	
011	2	02	STORE MAX RANGE CONSTANT	048	3	05	CONDITIONAL TEST OF GROSS WEIGHT CATEGORY INCREMENTS CONSTANTS
012	.	-62		049	÷	-24	
013	7	07		050	CHS	-22	
014	ST01	35 01		051	RTN	24	
015	XZY	-41	STORE LOITER CONSTANT	052	*LEL4	21 04	
016	2	02		053	XZY9	16-35	
017	2	02		054	GT01	22 01	
018	0	00		055	.	-62	
019	ST02	35 02	SET INITIAL GROSS WEIGHT BRACKET	056	0	00	
020	R+	-31		057	3	02	
021	1	01		058	ST-1	35-45 01	
022	2	02		059	R4	-31	
023	7	07		060	5	05	
024	.	-62		061	ST-2	35-45 02	
025	5	05		062	-	-45	
026	GT04	22 04		063	GT05	22 05	
027	*LEL5	21 12	COMPUTE LOITER IAS	064	RTN	24	
028	6566	23 11		065	*LEL5	21 05	
029	ROL2	36 02		066	XZY9	16-35	
030	FTN	24		067	GT01	22 01	
031	*LEL1	21 01		068	.	-62	
032	6567	23 07		069	0	00	
033	R4	-31		070	2	02	
034	1	01	CHECK ALTITUDE RESTRICTIONS	071	ST-1	35-45 01	
035	1	01		072	R+	-31	
036	7	07		073	5	05	
037	.	-62		074	ST-2	35-45 02	

075	-	-45
076	GT04	22 04
077	RTN	24
078	*LEL6	21 06
079	RCL0	36 08
080	2	02
081	8	08
082	EEA	-23
083	3	03
084	XYY9	16-34
085	FTN	24
086	XZY	-41
087	FFTX	-14
088	GT0C	22 13
089	*LEL7	21 07
090	R4	-31
091	7	07
092	2	02
093	.	-62
094	5	05
095	XZY9	16-35
096	FTN	24
097	XZY	-41
098	PRTX	-14
099	GT0C	22 13
100	RTN	24
101	R/S	51

CHECKS ALTITUDE IF ILLEGAL,

ERROR DISPLAY

CHECKS GROSS WEIGHT CATEGORY

IF ILLEGAL, ERROR DISPLAY

E. Computational Analysis

Using the HP-67 standard curve fitting program, a good linear fit was obtained on the four engine maximum range data. There is a linear relationship between altitude and indicated airspeed for each gross weight category. The coefficient of determination was equal to 1.00 in all cases, indicating a good fit. The following equations were used; loiter airspeeds are constant for each category.

<u>G.W. (1000 lbs)</u>	<u>Max range IAS</u>	<u>Loiter IAS</u>
132.5-127.5	$y = 270 - x/1000$	220
127.5-122.5	$y = 267 - x/1000$	215
122.5-117.5	$y = 265 - x/1000$	210
117.5-112.5	$y = 262 - x/1000$	205
112.5-107.5	$y = 260 - x/1000$	200
107.5-102.5	$y = 257 - x/1000$	195
102.5- 97.5	$y = 255 - x/1000$	190
97.5- 92.5	$y = 252 - x/1000$	185
92.5- 87.5	$y = 250 - x/1000$	180
87.5- 82.5	$y = 247 - x/1000$	175
82.5- 77.5	$y = 245 - x/1000$	170
77.5- 72.5	$y = 242 - x/1000$	165

x = altitude in feet and y = maximum range IAS.

V. USER-CONTROLLED SIMULATION OF APPROACH AND LANDING FOR
THE P-3 AIRCRAFT by LT J. Aiken

A. Problem Statement

This program simulates an aircraft approach and landing. Specifically, it is a time-step simulation of the final five miles of a precision approach for a Lockheed P-3 ORION aircraft. The simulation is user-controlled which allows the user to act as pilot and make the decisions which control the movement of the airplane during its final approach phase. The purpose of the program is to simulate accurately the flight of the aircraft and display to the operator his rate of movement and position resulting from his manipulation of the controls.

B. Operation Analysis

Relevant information on the airfield is as follows:

Runway	8000 ft (length 200 ft (width) 180 degrees magnetic heading SEA LEVEL elevation
Approach	TOUCHDOWN POINT 1000ft beyond approach threshold GLIDE SLOPE 2.83 degrees 2 min 18 sec time required at 135 kts ground speed FINAL APPROACH FIX: 5 miles, 1500ft (starting point)

The aircraft weighs 90,000 lbs with approach speeds 135/121 kts (approach flaps/land flaps). Note however that no provision is made for changing the gear/flap configuration so it is essentially an "approach-flap" landing. The simulation starts with the aircraft in motion: 1500 feet MSL, 135 kts IAS, 650 ft/min descent rate, landing gear down and approach flaps. The simulation allows the user to select horsepower settings, nose attitude, heading, wind direction, wind velocity, and time interval. At the end of a time interval the simulation is halted and the critical flight information is displayed to the operator, allowing him to alter controlling parameters and continue the flight. The simulation continues in this manner until the aircraft lands. Vital landing parameters are displayed and the simulation is complete. The simulation realistically responds to control changes provided the aircraft is flown in a somewhat reasonable fashion. Extreme deviations and maneuvers other than those required during an approach are not designed into the program.

C. Computational Algorithm

1. Initialize the aircraft at the starting point.
2. Input time step, wind direction and wind velocity.
3. Input horsepower, nose attitude, heading, and number of time steps desired.
4. Compute course deviation.
5. Compute horizontal acceleration.
6. Compute vertical acceleration.
7. Compute final velocity and average vertical velocity.
8. Compute altitude.
9. Compute final and average horizontal velocity.
10. Compute distance remaining based on ground speed.
11. Compute glide slope height and deviation from glide slope.
12. Check altitude less than 0.
13. DSZ (number of time steps is the counter) GTO 4 above.
14. Display approach parameters after completing desired time steps.
15. Display landing parameters upon landing.
16. Clear primary and secondary registers, GTO 1 for new problem.

D. HP-67 Calculator Program

1. User Instructions

Step	Instruction	Input	Key (s)	Output
1.	Enter program card			
2.	Clear primary and secondary registers			
3.	Initialize		f e	30384
4.	Enter time step	seconds	STO C	
5.	Enter wind direction	degrees	STO D	
6.	Enter wind velocity	knots	STO E	
7.	Enter horsepower	HP	A	HP
8.	Enter nose attitude	<u>+</u> degrees	B	Nose attitude
9.	Enter heading	degress	C	Heading
10.	Enter number of time steps	integer	D	flashing
	<u>Output</u>			
	Altitude. Airspeed (Packed)			Alt. airspeed
	Descent rate (ft/min)		R/S	Descent rate
	Above (+) or below (-) glide slope		R/S	Feet hi/lo
	Distance to go Airborne (miles) Landed (feet)		R/S	<u>+</u> Distance to go
	Right (+) or left (-) of of course		R/S	ft right/left
11.	If altitude GT zero go to Step 4; make new entries only if change desired. Step 10 must be re-entered.			

2. Sample Problem

Input							Output					
Time Step	Wind Direction	Wind Velocity	Horsepower	Nose Attitude	Heading	Steps	Altitude	Airspeed	Descent Rate	Above/Below Glide Slope	Distance To Go	Right/Left of Course
20	210	20	800	1.5	185	1	1295	133	-570	-1	4.4	63
					183	1	1108	132	-552	2.4	3.7	-34
			790		184	1	922	132	-560	6.9	3.1	-51
			775		185	1	732	132	-580	6.3	2.4	11
					184	1	538	132	-584	1.6	1.8	- 6
10			790		184	1	441	132	-575	0	1.5	-14
					184.8	1	346	132	-574	-1	1.2	9
			795	2	184.3	1	252	131	-555	-1	.85	13
5			700	3.5	184	1	205	129	-559	-1	.7	8
				4		1	159	127	-552	-2	.5	4
					184.1	1	113	127	-552	-2	.4	2
					4.3	184	67	127	-547	-3	.24	- 2
					4.8	1	22	126	-538	-3	.08	- 7
2				6		1	4	126	-530	-3	.02	- 8
				8	182.5	1	0	125	-516	-2	40	-22

Sample Problem Keystroke Sequence

635e		-580.08	***
36384.00	***		R/S
20.00	STCC	6.33	***
210.00	STCC		R/S
20.00	STDE	2.44	***
800.00	GSEH		R/S
1.50	GSEE	11.46	***
185.00	GSEC	184.00	GSEC
1.00	GSEC	1.00	GSEC
1255.135	***	536.132	***
	R/S		R/S
-570.00	***	-584.02	***
	R/S		R/S
-1.02	***	1.61	***
	R/S		R/S
4.35	***	1.80	***
	R/S		R/S
62.92	***		R/S
		-5.66	***
183.00	GSEC	10.00	STCC
1.00	GSEC	790.00	GSEH
1100.132	***	184.00	GSEC
	R/S	1.00	GSEC
-552.00	***	441.132	***
	R/S		R/S
2.42	***	-575.41	***
	R/S		R/S
3.71	***	-0.35	***
	R/S		R/S
-34.34	***	1.49	***
790.00	GSEH		R/S
184.00	GSEC	-14.22	***
1.00	GSEC	184.00	GSEC
922.132	***	1.00	GSEC
	R/S	346.132	***
-560.40	***		R/S
	R/S	-573.69	***
6.89	***		R/S
	R/S	-1.40	***
3.63	***		R/S
	R/S	1.17	***
-51.46	***		R/S
775.00	GSEH	9.24	***
185.00	GSEC	795.00	GSEH
1.00	GSEC	2.00	GSEE
		184.30	GSEC
732.132	***	1.00	GSEC
	R/S	252.131	***

Sample Problem Keystroke Sequence (cont.)

	R/S		1.00 GSEC
-555.34 ***	R/S	67.127 ***	R/S
-1.25 ***	R/S	-546.74 ***	R/S
0.85 ***	R/S	-2.72 ***	R/S
12.65 ***	R/S	0.24 ***	R/S
5.00 STOC		-2.43 ***	R/S
700.00 GSEC		4.80 GSEC	
3.50 GSEC		1.00 GSEC	
184.00 GSEC		22.126 ***	
1.00 GSEC		R/S	
205.129 ***	R/S	-538.29 ***	R/S
-559.51 ***	R/S	-2.76 ***	R/S
-1.27 ***	R/S	0.08 ***	R/S
0.69 ***	R/S	-6.71 ***	
8.41 ***		2.00 STOC	
4.00 GSEC		6.00 GSEC	
1.00 GSEC		1.00 GSEC	
159.127 ***	R/S	4.126 ***	R/S
-552.84 ***	R/S	-530.41 ***	R/S
-1.85 ***	R/S	-2.61 ***	R/S
0.54 ***	R/S	0.02 ***	R/S
4.13 ***		-8.42 ***	
184.10 GSEC		8.00 GSEC	
1.00 GSEC		182.50 GSEC	
113.127 ***	R/S	1.00 GSEC	
-551.51 ***	R/S	0.185 ***	R/S
-2.35 ***	R/S	-516.84 ***	R/S
0.35 ***	R/S	-2.24 ***	R/S
1.85 ***		40.50 ***	R/S
4.30 GSEC		-22.16 ***	
184.00 GSEC			

3. Program Storage Allocations and Program Listing

Registers:

R0: Right/left of course	S1: Horizontal force
R1: Horsepower	S2: Vertical force
R2: Nose attitude	
R3: Horizontal velocity	
R4: Vertical velocity	
R5: Altitude	
R6: Horizontal acceleration	
R7: Vertical acceleration	
R8: Distance remaining	
R9: Glide Slope Altitude	
RA: DELH	
RB: Heading	
RC: Time step	
RD: Wind direction	
RE: Wind velocity	
RI: Number of time steps	

Initial Flag Status:

0: OFF, Unused	2: OFF, Set ON upon landing
1: OFF, Unused	3: OFF, Unused

User Control Keys:

A: Horsepower	a:
B: Nose attitude	b:
C: Heading	c:
D: Time steps, start computation	d:
E:	e: Initialize

001	*LELA	21 11	INPUT	061	GS81	23 01	COMPUTE
002	ST01	35 01		062	ST06	35 06	HORIZONTAL ACCEL
003	R/S	51		063	GS82	23 02	COMPUTE
004	*LELB	21 12		064	ST07	35 07	VERTICAL ACCEL
005	ST02	35 02		065	RCL4	36 04	COMPUTE
006	R/S	51		066	RCL7	36 07	
007	*LELC	21 13		067	RCL0	36 13	
008	ST06	35 12		068	X	-35	VERTICAL
009	R/S	51		069	ST+4	35-55 04	VELOCITY
010	*LELe	21 16 15		070	XZY	-41	
011	2	02	INITIALIZE	071	RCL4	36 04	
012	2	02		072	+	-55	COMPUTE
013	8	02		073	2	02	
014	ST03	35 03		074	+	-24	
015	1	01		075	RCL0	36 13	ALTITUDE
016	1	01		076	X	-35	
017	CHS	-22		077	ST+5	35-55 05	
018	ST04	35 04		078	RCL3	36 03	COMPUTE
019	1	01		079	RCL6	36 06	HORIZONTAL
020	5	05		080	RCL0	36 13	
021	0	00	SPEEDS	081	X	-35	
022	0	00		082	ST+3	35-55 03	VELOCITY
023	ST05	35 05		083	XZY	-41	
024	3	03		084	RCL3	36 03	
025	0	00		085	+	-55	COMPUTE
026	3	03		086	2	02	
027	6	06		087	+	-24	
028	4	04		088	RCL0	36 14	GROUND
029	ST08	35 02		089	1	01	
030	R/S	51		090	8	08	
031	*LELD	21 14	ALTITUDE	091	0	00	SPEED
032	ST01	35 45		092	-	-45	
033	*LELE	21 05		093	ABS	16 31	
034	RCL6	36 12		094	COS	42	AND
035	1	01		095	RCL6	36 15	
036	8	08		096	1	01	
037	0	00		097	.	-62	DISTANCE
038	-	-45		098	6	06	
039	SIN	41		099	9	09	
040	2	02	COMPUTE	100	X	-35	TO GO
041	3	03		101	X	-35	
042	0	00		102	-	-45	
043	X	-35		103	RCL0	36 13	COMPUTE
044	1	01		104	X	-35	
045	8	08		105	CHS	-22	
046	0	00		106	RCL8	36 08	GLIDE-SLOPE
047	RCL0	36 14		107	+	-55	
048	-	-45		108	ST08	35 08	
049	SIN	41		109	.	-62	HEIGHT
050	RCL6	36 15		110	0	00	
051	1	01	COURSE	111	4	04	
052	.	-62		112	9	09	AND
053	6	06		113	X	-35	
054	9	09		114	ST09	35 09	
055	X	-35		115	CHS	-22	FROM
056	X	-35		116	RCL5	36 05	
057	+	-55		117	+	-55	
058	RCL0	36 13		118	ST04	35 11	GLIDE-SLOPE
059	X	-35		119	RCL5	36 05	
060	ST+0	35-55 00		120	X 02	16-45	
			DEVIATION	121	GT09	22 09	CHECK ALT < 0
				122	DECI	16 25 46	
				123	GT08	22 08	
							LOOP CONTROL

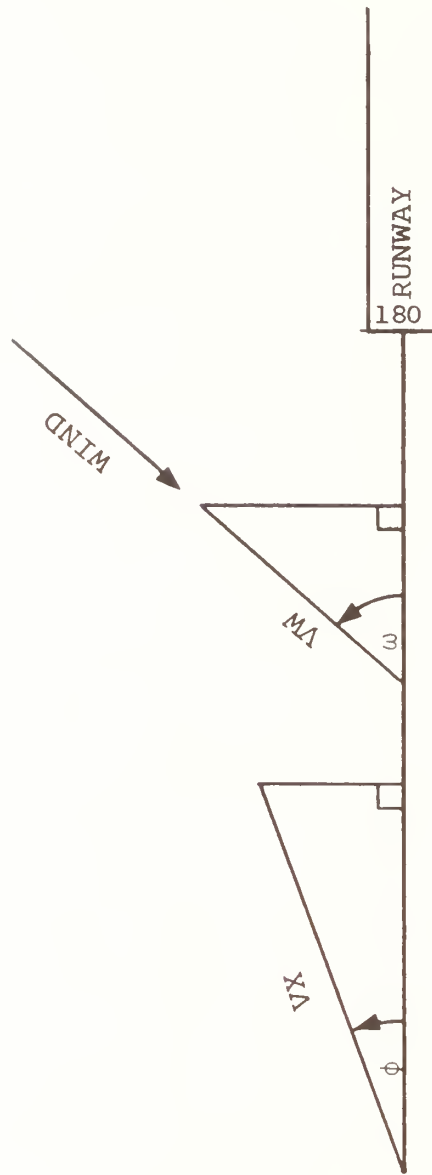
124	*LEL7	21	07	ALTITUDE DECIMAL AIRSPEED (PACKED)	169	*LEL1	21	01	COMPUTE HORIZONTAL FORCE
125	FCL5	36	05		170	FCL2	36	02	
126	INT	16	34		171	.	-62		
127	FCL3	36	03		172	4	04		
128	1	01			173	CH5	-22		
129	6	06		RATE OF DESCENT	174	A	-35		COMPUTE HORIZONTAL ACCELERATION
130	8	08			175	EEN	-23		
131	8	08			176	3	03		
132	÷	-24			177	CH5	-22		
133	+	-55			178	FCL1	36	01	
134	DSP3	-63	03	HIGH/LOW	179	X	-35		COMPUTE HORIZONTAL ACCELERATION
135	R/S	51			180	+	-55		
136	DSP2	-63	02		181	.	-62		
137	RCL4	36	04		182	4	04		
138	6	06			183	-	-45		
139	0	00		DISTANCE TO GO (FEET IF LANDED)	184	F25	16-51		COMPUTE VERTICAL FORCE
140	X	-35			185	RCL1	36	01	
141	R/S	51			186	XZY	-41		
142	RCLA	36	11		187	ST01	35	01	
143	R/S	51			188	XZY	-41		
144	RCL8	36	08	LEFT/RIGHT	189	-	-45		COMPUTE VERTICAL FORCE
145	F29	16	23		190	F25	16-51		
146	GT05	22	05		191	RCL6	36	06	
147	6	06			192	5	05		
148	0	00			193	÷	-24		
149	7	07		LANDED... ADJUST DISTANCE TO ZERO ALT.	194	+	-55		COMPUTE VERTICAL FORCE
150	7	07			195	RTN	24		
151	÷	-24			196	*LEL2	21	02	
152	*LEL5	21	05		197	.	-62		
153	R/S	51			198	0	00		
154	RCL0	36	00	SET ALT = 0 SET FLAG 2	199	0	00		COMPUTE VERTICAL FORCE
155	R/S	51			200	1	01		
156	*LEL9	21	09		201	RCL1	36	01	
157	RCL8	36	08		202	X	-35		
158	RCL5	36	05		203	.	-62		
159	CH5	-22		SET ALT = 0 SET FLAG 2	204	8	08		COMPUTE VERTICAL FORCE
160	2	02			205	-	-45		
161	0	00			206	RCL2	36	02	
162	X	-35			207	.	-62		
163	+	-55			208	0	00		
164	ST02	35	02	SET ALT = 0 SET FLAG 2	209	5	05		COMPUTE VERTICAL FORCE
165	0	00			210	X	-35		
166	STC5	35	05		211	+	-55		
167	SF2	16	21		212	F25	16-51		
168	GT07	22	07		213	RCL3	36	03	
					214	XZY	-41		COMPUTE VERTICAL FORCE
					215	ST02	35	02	
					216	XZY	-41		
					217	-	-45		
					218	F25	16-51		
					219	RCL7	36	07	COMPUTE VERTICAL FORCE
					220	5	05		
					221	÷	-24		
					222	+	-55		
					223	RTN	24		
					224	R/S	51		

E. Mathematical Analysis

The variable names used are:

VX	Horizontal Velocity
VY	Vertical Velocity
VW	Wind Velocity
DRA	Drift Rate Due to A/C HDG
DRW	Drift Rate Due to Wind
TDR	Total Drift Rate
GS	Ground Speed
AX	Horizontal Velocity
AY	Vertical Velocity
D	Distance to go
H	Glide Slope Altitude
DELH	Deviation from Glide Slope
ALT	Altitude
t	Time Step

COURSE DEVIATION COMPUTATION



$$\phi = \text{HDG} - 180$$

$$\text{DRA} = VX \sin \phi$$

$$\omega = 180 - \text{Wind Dir}$$

$$\text{DRW} = VW \sin \omega$$

$$\phi \text{ is negative (on this diagram)}$$

$$\text{TRD} = \text{DRA} + \text{DRW}$$

$$\text{TOTAL DRIFT} = \text{TDR}(t)$$

DISTANCE, GROUNDSPED, GLIDE SLOPE

$$\text{Avg HOR Velocity} = \frac{1}{2} (VX_0 + VX_1) \quad (\text{for one time step})$$

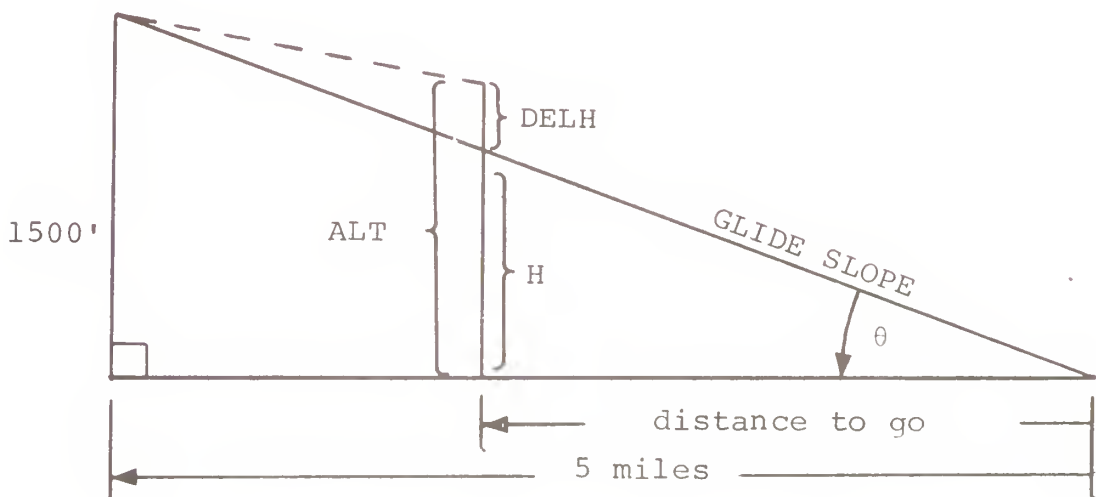
$$\text{Distance travelled} = GS(t)$$

$$GS = VX - VW \cos|\text{wind dir} - 180|$$

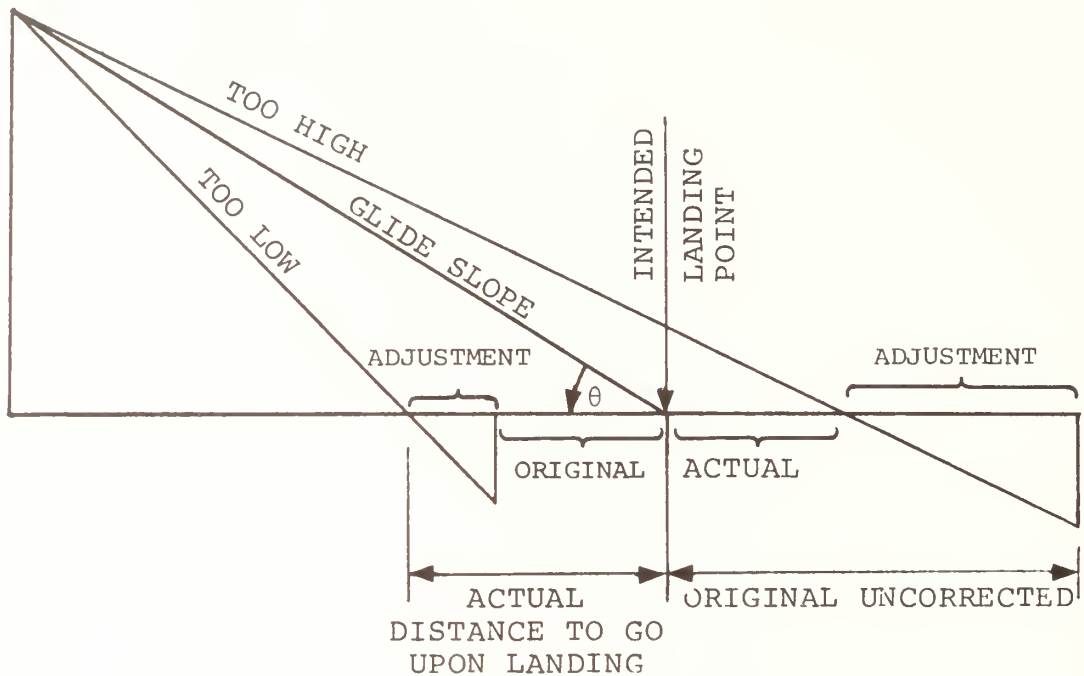
$$D = \text{prior remaining} - \text{distance travelled}$$

$$H = D \tan \theta$$

$$\text{DELH} = \text{ALT} - H$$



LANDING DISTANCE ADJUSTMENT



Two situations are illustrated above.

1. Aircraft above glide slope (lands long).
2. Aircraft below glide slope (lands short).

An adjustment is required because the aircraft is allowed to descend until the end of the time step. On the landing time step the aircraft will have descended below zero altitude.

The adjustment in either case is: $\text{adjustment} = H / (\tan \theta)$.

FORCES, ACCELERATIONS, AND VELOCITIES

$FX = -\theta + .001HP - .4$	FX = horizontal force
$FY = .001HP - .8 + .05\theta$	FY = vertical force
$AX' = (FX^1 - FX^0) + .2AX^0$	θ = nose attitude
$AY' = (FY^1 - FY^0) + .2AY^0$	Superscripts denote different time steps
$VX_1 = VX_0 + AX(t)$	Subscripts denote start and end of a time step.
$VY_1 = VY_0 + AY(t)$	

Although the author is not an aeronautical engineer, it was felt that his understanding of the basic laws of physics complemented by considerable pilot experience would serve sufficiently to accomplish the goals of this project. The formulae for force and acceleration were arrived at after testing several trial formulae on experienced P-3 pilots. The unanimous opinion was that the current program enables the simulation to closely model the actual flight characteristics of the aircraft.

VI. FLIGHT CREW MANAGEMENT USING THE HP-97 by LT Kenneth W. Peters

A. Problem Statement

A flight crew's most recent landing day and time is known. Using requirements for crew rest and postflight and preflight duration, compute when the flight crew will be available for take-off again. For planning and scheduling purposes, list crews in order of availability. For required onstation times compute takeoff, onstation, offstation and landing times for a given number of flights. Determine if flight crews will be available to meet this schedule.

B. Operational Analysis

When planning an operation requiring scheduling of several flight crews, crew availability must be considered. Accurate and easily understood crew records are necessary to meet both operational and safety requirements.

C. Computational Algorithm

1. Flight crew availability

- a. Enter required postflight to preflight crew rest time.
- b. Enter crew number and their most recent landing day and time.
- c. Compute the crew's earliest possible takeoff day and time using one hour for postflight and three hours for preflight.

2. List crews in order of availability
 - a. Enter number of flight crews = N.
 - b. Compare crew N availability with crew (N-1) availability. Store crew number and availability of crew which can take off soonest. Compare this crew with crew (N-x), (x = 1,2,3,4,...,(N-1)), and store the crew which is available the soonest.
 - c. When most available crew has been determined, add 10,000 days to its takeoff availability date and increment counter.
 - d. Repeat Steps b and c until all crews have been listed, i.e. counter = N.
 - e. Restore crew availability data by subtracting 10,000 days from each crew's availability date.
3. Operational schedule.
 - a. Enter required number of flights from a particular base and the gap (+) desired for onstation coverage.
 - b. Compute takeoff day and time, onstation day and time, offstation day and time, and landing day and time.
 - c. Check number of flights versus counter. Repeat b as required.
 - d. Compare print-out of required takeoff times with crew availability listing (see Part 2 above).

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Compute flight crew availability			
a.	Enter crew rest time	HH.MM	fA	
b.	Enter crew rest data:			
	Crew #	$1 \leq \# \leq 19$	↑	
	Postflight time	HH.MM	↑	
	Preflight time	HH.MM	↑	
	Landing Julian day and time	DAY.HHMM	A	Crew # Availability DAY.HHM
	NOTE: If change in year will occur, Julian date can be entered as: YRDAY.HHMM			
3.	List crews in order of availability. Enter # of crews = N	$1 \leq N \leq 19$	B	Crew # Availability DAY.HHMM (repeated N times)
4.	Compute operational flight schedule			
a.	Enter # of flights and gap in onstation coverage: # of flights = N	$1 \leq N \leq 25$	↑	
	gap in coverage	HH.MM	fC	
b.	Compute schedule			
	onstation day and time	DAY.HHMM	↑	
	one-way transit time	HH.MM	↑	
	mission time	HH.MM	C	Flight # Takeoff DAY.HHMM Onsta DAY.HHMM Offsta DAY.HHMM Land DAY.HHMM
5.	(Optional) Store crew availability data on magnetic card: Set W/PRGM-RUN switch to RUN To reload data: Set W/PRGM-RUN switch to RUN and load data card obtained above		f W/DATA	

2. Sample Problem

a. Input the following data:

Crew rest = 15 hours

<u>Crew #</u>	<u>Postflight</u>	<u>Preflight</u>	<u>Landing</u>
1	1	3	2.2330
2	1	3	1.1200
3	1	3	2.1800
4	1	3	1.1532

b. Compute crew availability.

Answer:

<u>Crew #</u>	<u>Availability</u>
1	3.1830
2	2.0700
3	3.1300
4	2.1032

c. Input the following crew status:

<u>Crew #</u>	<u>(in) Register #</u>	<u>Availability</u>
1	1	3.1830
2	2	2.0700
3	3	3.1300
4	4	2.1032
5	5	2.0958
6	6	2.2020
7	7	3.1930
8	8	5.0730
9	9	4.0930
10	10	6.0430
11	11	4.0345
12	12	2.0730
13	13	8.0130
14	14	5.0315
15	15	3.0545
16	16	3.1300
17	17	9.0955
18	18	2.1030

d. Output listing of crews in order of availability.

Answer: 2, 12, 5, 18, 4, 6, 15, 3, 16, 1, 7, 11, 9,
14, 8, 10, 13, 17.

e. Develop an operational flight schedule consisting of six (6) flights with a zero (0) gap in onstation coverage. The first onstation time is 2.1830, it takes one (1) hour and fifty (50) minutes for a one-way transit, and a flight from this base has nine (9) hours of total mission time.

Answer:

<u>Flight #</u>	<u>Takeoff</u>	<u>Onsta</u>	<u>Offsta</u>	<u>Land</u>
1	2.1640	2.1830	2.2350	3.0140
2	2.2200	2.2350	3.0510	3.0700
3	3.0320	3.0510	3.1030	3.1220
4	3.0840	3.1030	3.1550	3.1740
5	3.1400	3.1550	3.2110	3.2300
6	3.1920	3.2110	4.0230	4.0420

f. Develop an operational flight schedule consisting of two (2) flights with a two (2) hour gap in onstation coverage. The first onstation time is 2.1830, with the same transit and mission time as in (e).

✖ Answer:

<u>Flight #</u>	<u>Takeoff</u>	<u>Onsta</u>	<u>Offsta</u>	<u>Land</u>
1	2.1640	2.1830	2.2350	3.0140
2	3.0000	3.0150	3.0710	3.0900

- g. Develop an operational flight schedule with the same parameters as in (f), except with a negative two (-2) hour gap in onstation coverage.

Answer:

<u>Flight #</u>	<u>Takeoff</u>	<u>Onsta</u>	<u>Offsta</u>	<u>Land</u>
1	2.1640	2.1830	2.2350	3.0140
2	2.2100	2.2250	3.0410	3.0600

- h. With one flight from Base A with the first onstation time as above and one flight from Base B to the same operational area, compute the flight schedule. One-way transit time from Base B is two (2) hours, but mission time is now ten (10) hours. Zero (0) gap in coverage is desired.

Answer:

<u>Flight #</u>	<u>Takeoff</u>	<u>Onsta</u>	<u>Offsta</u>	<u>Land</u>
1	2.1640	2.1830	2.2350	3.0140
2	2.2120	2.2350	3.0450	3.0720

- i. With four (4) flight crews with crew availability as follows:

<u>Crew #</u>	<u>Availability</u>
1	3.1830
2	2.0700
3	3.1300
4	3.1700

determine if they are available to meet an operational flight schedule with the following conditions:

4 flights

2 hour gap in onstation coverage

3.0100 is the first onstation time

1 hour and 30 minute one-way transit

10 hour mission time

Answer: No, a crew is not available to meet the second scheduled takeoff time.

Examples a, b

15.0000	GSEH
1.0000	ENT1
1.0000	ENT1
3.0000	ENT1
2.2730	GSEH
1.	***
3.1630	***
2.0000	ENT1
1.0000	ENT1
3.0000	ENT1
1.1200	GSEH
2.	***
2.0700	***

3.0000	ENT1
1.0000	ENT1
3.0000	ENT1
2.1000	GSEH
3.	***
3.1500	***
4.0000	ENT1
1.0000	ENT1
3.0000	ENT1
1.1532	GSEH
4.	***
2.1032	***

Examples c, d

16.0000 GSEH	15. ***	9. ***
2. ***	3.0545 ***	4.0930 ***
2.0700 ***		
12. ***	3. ***	14. ***
2.0730 ***	3.1300 ***	5.0315 ***
5. ***	16. ***	8. ***
2.0958 ***	3.1300 ***	5.0730 ***
18. ***	1. ***	10. ***
2.1030 ***	3.1630 ***	6.0430 ***
4. ***	7. ***	13. ***
2.1032 ***	3.1930 ***	9.0130 ***
6. ***	11. ***	17. ***
2.2020 ***	4.0345 ***	9.0955 ***

Example e

```

6.0000 ENT:
0.0000 GSEC:

2.1830 ENT:
1.5000 ENT:
9.0000 GSEC
1.000000000 ***
2.1640 ***
2.1830 ***
2.2350 ***
3.0140 ***

2.000000000 ***
2.2200 ***
2.2350 ***
3.0510 ***
3.0700 ***

3.000000000 ***
3.0330 ***
3.0510 ***
3.1030 ***
3.1220 ***

4.000000000 ***
3.0040 ***
3.1030 ***
3.1550 ***
3.1740 ***

5.000000000 ***
3.1400 ***
3.1550 ***
3.2110 ***
3.2300 ***

6.000000000 ***
3.1920 ***
3.2110 ***
4.0230 ***
4.0420 ***

```

Example f

```

2.0000 ENT:
2.0000 GSEC:

2.1830 ENT:
1.5000 ENT:
9.0000 GSEC
1.000000000 ***
2.1640 ***
2.1830 ***
2.2350 ***
3.0140 ***

2.000000000 ***
3.0000 ***
3.0150 ***
3.0710 ***
3.0900 ***

```

Example g

```

2.0000 ENT:
-1.0000 GSEC:

2.1830 ENT:
1.5000 ENT:
9.0000 GSEC
1.000000000 ***
2.1640 ***
2.1830 ***
2.2350 ***
3.0140 ***

2.000000000 ***
2.2100 ***
2.2250 ***
3.0410 ***
3.0600 ***

```

Example h

```

1.0000 ENT?
0.0000 GSE:

3.1830 ENT?
1.5000 ENT?
9.0000 GSEC
1.000000000 ***
2.1640 ***
2.1630 ***
2.2350 ***
3.0140 ***

1.0000 ENT?
0.0000 GSEC

2.2350 ENT?
2.3000 ENT?
10.0000 GSEC
1.000000000 ***
2.2120 ***
2.2350 ***
3.0450 ***
3.0720 ***

```

Example i

```

4.0000 GSEB
2. ***
3.0700 ***

3. ***
3.1300 ***

4. ***
3.1700 ***

1. ***
3.1830 ***

```

```

4.0000 ENT?
2.0000 GSEC
3.0100 ENT?
1.3000 ENT?
10.0000 GSEC
1.000000000 ***
2.2330 ***
3.0100 ***
3.0800 ***
3.0930 ***

2.000000000 ***
3.0530 ***
3.1000 ***
3.1700 ***
3.1830 ***

3.000000000 ***
3.1730 ***
3.1900 ***
4.0200 ***
4.0330 ***

4.000000000 ***
4.0230 ***
4.0400 ***
4.1100 ***
4.1230 ***

```

3. Program Storage Allocation and Listing

Registers:

R0: Crew rest	S0: Crew 10 availability
R1: Crew 1 availability	S1: Crew 11 availability
R2: Crew 2 availability	S2: Crew 12 availability
R3: Crew 3 availability	S3: Crew 13 availability
R4: Crew 4 availability	S4: Crew 14 availability
R5: Crew 5 availability	S5: Crew 15 availability
R6: Crew 6 availability	S6: Crew 16 availability
R7: Crew 7 availability	S7: Crew 17 availability
R8: Crew 8 availability	S8: Crew 18 availability
R9: Crew 9 availability	S9: Crew 19 availability
RA: Landing data; # of crews	
RB: First onsta time; crew #; takeoff time	
RC: Mission time; flight counter	
RD: Gap between onstation periods; ith availability	
RE: Flight counter; crew counter; one-way transit	

Initial Flag Status and Use

0: Unused	2: OFF, day correction
1: Unused	d: OFF, hour correction

User Controlled Keys

A: Crew # ↑, postflight ↑, preflight ↑, land = compute availability	
B: crew # ↑; compute listing	
C: onsta ↑, one-way transit ↑, mission time = flight schedule	
D: unused	
E: unused	
a: crew rest	d: unused
b: unused	e: unused
c: flight ↑, gap	

001	*LELc	21 16 13	Input and Store # of flights and desired gap
002	STOD	35 14	
003	R↓	-31	
004	STOA	35 11	
005	RTN	24	
006	*LELC	21 13	
007	CF2	16 22 03	Compute flight schedule
008	CF3	16 22 03	Input and store onstation day and time
009	STOC	35 13	Input and store one-way transit time
010	R↓	-31	Input and store total mission time
011	STOE	35 15	
012	R↓	-31	
013	STOB	35 12	
014	GSE0	23 00	
015	RCLE	36 15	Unpack ddd.hhmm to hh.mm
016	CHS	-22	
017	HMS+	16-55	Compute takeoff time
018	X10?	16-45	Check for time greater than 24 hours
019	GSEe	23 16 15	
020	EEX	-23	
021	2	02	
022	÷	-24	
023	RCLB	36 12	
024	INT	16 34	
025	F3?	16 23 03	
026	GSEd	23 16 14	
027	+	-55	Takeoff ddd.hhmm
028	STOB	35 12	
029	1	01	
030	*LEL9	21 09	
031	STOI	35 46	
032	DSF0	-63 06	-----
033	FFTX	-14	Start loop
034	RCL6	36 12	Print flight #
035	F3?	16 23 02	
036	GSE2	23 02	
037	DSF4	-63 04	
038	FRTX	-14	Print takeoff ddd.hhmm
039	STOB	35 12	
040	STOB	35 00	
041	GSE0	23 00	
042	RCLE	36 15	
043	HMS+	16-55	Compute and print onstation ddd.hhmm
044	GSE0	23 14	
045	GSEe	23 15	
046	GSE0	23 00	
047	RCLE	36 15	
048	RCLE	36 15	
049	HMS+	16-55	
050	CHS	-22	
051	RCLC	36 13	
052	HMS+	16-55	
053	HMS+	16-55	Compute and print offstation ddd.hhmm
054	GSE0	23 14	
055	GSEe	23 15	
056	GSE0	23 00	
057	RCLE	36 15	
058	HMS+	16-55	Compute and print landing ddd.hhmm
059	GSE0	23 14	
060	GSEe	23 15	
061	SPC	16-11	

062	RCLE	36 12
063	GSE0	23 00
064	RCLE	36 12
065	HMS+	16-55
066	RCLE	36 12
067	RCLE	36 12
068	HMS+	16-55
069	CHS	-22
070	HMS+	16-55
071	RCLD	36 14
072	HMS+	16-55
073	GSE0	23 14
074	RCLB	36 12
075	INT	16 34
076	+	-55
077	ST08	35 12
078	RCLA	36 11
079	1	01
080	+	-55
081	1	01
082	RCLI	36 45
083	+	-55
084	X=Y?	16-33
085	R/S	51
086	GT09	22 09
087	*LBLD	21 14
088	2	02
089	4	04
090	X≠Y?	16-35
091	GSE8	23 08
092	X≠Y	-41
093	EEX	-23
094	2	02
095	÷	-24
096	RTN	24
097	*LBL8	21 08
098	CHS	-22
099	HMS+	16-55
100	SF2	16 21 02
101	X≠Y	-41
102	RTN	24
103	*LBL5	21 15
104	RCL0	36 00
105	INT	16 34
106	F2?	16 23 02
107	GSE2	23 02
108	+	-55
109	PRTX	-14
110	ST00	35 00
111	RTN	24
112	*LBL2	21 02
113	1	01
114	+	-55
115	RTN	24
116	*LBL0	21 00
117	FPC	16 44
118	EEX	-23
119	2	02
120	X	-35

Compute next takeoff ddd.hhmm

Increase counter

Check for exit from loop

Subroutine to correct for time greater
than 24 hours

Subroutine to correct time

Output subroutine
Print onstation, offstation, and land

Subroutine to correct date

Subroutine to change .hhmm to hh.mm

121	RTN	24	Subroutine to correct for negative time
122	*LELe	21 16 15	
123	2	02	
124	4	04	
125	HME+	16-55	
126	EF3	16 21 03	
127	RTN	24	
128	*LELd	21 16 14	
129	1	01	Subroutine to correct date
130	-	-45	
131	RTN	24	
132	*LELo	21 16 11	Store crew postflight to preflight rest time
133	ST00	35 00	
134	CF2	16 22 02	
135	RTN	24	
136	*LELH	21 11	
137	ST0A	35 11	Compute crew's earliest possible takeoff
138	P1	16-31	Store latest landing: ddd.hhmm
139	STOI	35 46	Store crew #
140	R4	-31	
141	FRC	16 44	
142	EEX	-23	
143	2	02	
144	x	-35	
145	HMS+	16-55	
146	HMS+	16-55	
147	RCL0	36 00	
148	HMS+	16-55	
149	GSE0	23 14	Compute crew I's takeoff availability
150	STOI	35 45	
151	RCLH	36 11	
152	INT	16 34	
153	F22	16 23 02	
154	GSE2	23 02	
155	ST+1	35-55 45	
156	RCL1	36 46	
157	DSP0	-63 00	
158	FRTX	-14	Print crew #
159	PCL1	36 45	
160	DSP4	-63 04	
161	PFTY	-14	Print crew's ddd.hhmm takeoff availability
162	SFC	16-11	
163	RTN	24	

164	*LBL6	21 12	List flight crews in order of availability
165	STOA	35 11	Store # of crews = N
166	0	00	Zero counter
167	STOC	35 13	
168	*LBL7	21 07	-----
169	RCLA	36 11	Begin loop
170	STOI	35 46	
171	STOB	35 12	
172	RCLi	36 45	Recall crew # N availability
173	STOD	35 14	
174	*LBL6	21 16 12	-----
175	DSZI	16 25 46	Recall crew # N-1 availability
176	GT01	22 01	
177	GT04	22 04	
178	*LBL1	21 01	-----
179	RCLD	36 14	Compare availability
180	RCLi	36 45	
181	X=Y?	16-34	
182	GSB3	23 03	
183	STOD	35 14	Store earliest ddd.hhmm
184	RCLI	36 46	
185	STOB	35 12	
186	GT06	22 16 12	
187	*LBL3	21 03	-----
188	X=Y	-41	Swap registers
189	STOD	35 14	
190	GT06	22 16 12	
191	*LBL4	21 04	-----
192	RCLB	36 12	Output routine
193	DSP0	-63 00	Print crew #
194	PRTX	-14	
195	STOI	35 46	
196	DSP4	-63 04	
197	RCLi	36 45	Print crew availability ddd.hhmm
198	PRTX	-14	
199	SPC	16-11	
200	SPC	16-11	
201	SPC	16-11	
202	SPC	16-11	
203	EEX	-23	
204	4	04	
205	ST+i	35-55 45	Add 1000 to day--i.e. create an artificially large date
206	1	01	
207	RCLC	36 13	
208	+	-55	
209	STOC	35 13	
210	RCLA	36 11	
211	X=Y?	16-33	Check for end of loop
212	GT05	22 05	
213	GT07	22 07	
214	*LBL5	21 05	-----
215	STOI	35 46	
216	*LBL6	21 06	-----
217	EEX	-23	Subtract 10,000 from date--i.e. correct artificially
218	4	04	
219	ST-i	35-45 45	
220	DSZI	16 25 46	
221	GT06	22 06	
222	R/S	51	

VII. TARGET MOTION ANALYSIS (TMA) OF A BEARINGS-ONLY TARGET
FROM A MOVING PLATFORM by LT P. W. Marzluff and
LT R. C. Pilcher

A. Problem Statement

Bearings to a target either stationary or moving with constant course and speed are available from a non-stationary tracking platform. Determine the target's range, course and speed.

B. Operational Analysis

The four bearing TMA technique used in this program requires a minimum of four target bearing observations taken during a minimum of two tracking legs. A target bearing observation must be made and entered for the time corresponding to the initiation of own ship course or speed change. Exact target bearing observations of 090° and 270° require the addition of 0.1° to the observed value to avoid infinite computational values. When the tracking problem carries into a new day, the previous day's time scale must be continued (i.e. a time of 0010 on the second day must be entered as 2410).

Own ship and target information is entered on card 1. Estimation of target parameters begins on card 1 and is completed on card 2. Entering supplemental target or own ship information and generating a new estimate again requires the use of card 1 and then card 2.

The accuracy of the estimates are dependent on the accuracy of the inputs, principally the target bearing, the magnitude

of course or speed change, and the number of observations made. For a more complete examination of the character of the estimates, see Reference 1.

C. Computational Algorithm

1. Input own ship's course and speed. Calculate and store velocity components.
2. Input time, t_i , and observed target bearing, B_i . Calculate elapsed time since the first observation, Δt_i , and $\tan B_i$. Calculate and store the matrix values.
3. When own ship changes course or speed enter t_i and B_i observed at the time the course or speed change was made. Enter new own ship course and speed prior to entering the next bearing observation.
4. When at least four target bearing observations have been entered (bearings taken on a minimum of two tracking legs), estimation of target range, course, and speed can be made.
5. Additional target and own ship information can be entered and new estimates made.

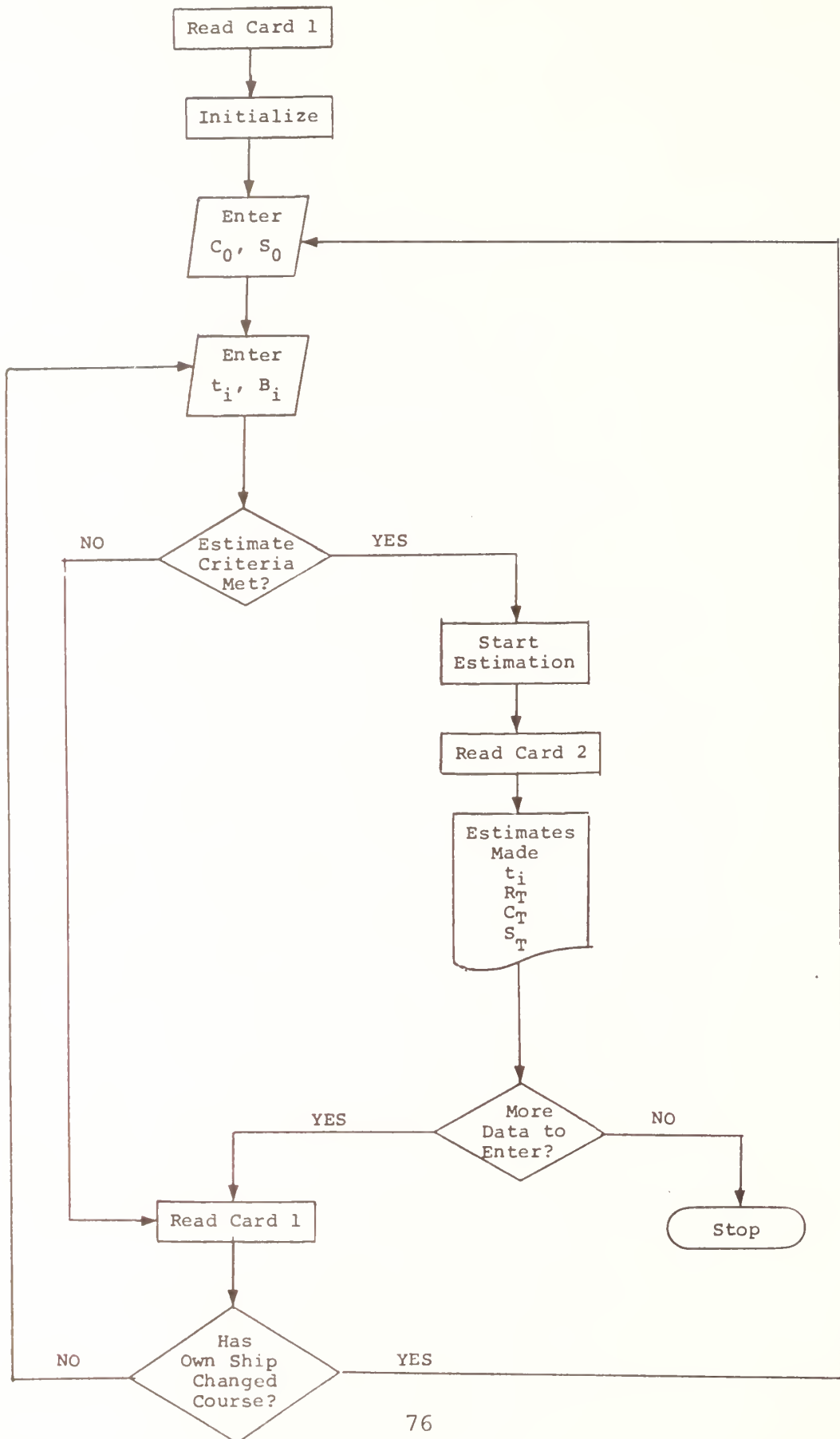
D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card 1			
2.	Initialize		fe	0.00
3.	Enter own ship course and speed	C_0 (degrees) S_0 (knots)	\uparrow fa	\dot{W}
4.	Enter the time and the observed bearing	t_i (HH.MM) B_i (degrees)	\uparrow C	i
5.	If estimation criteria is met go to Step 8; other- wise continue.			
6.	If own ship has changed course or speed go to Step 3; otherwise continue.			
7.	Go to Step 4.			
8.	Compute target range A. Start calculation B. Enter program Card 2 C. Continue calculation		E A	$t_i; R_T$ (yds)
9.	Compute target course		C	C_T (degrees)
10.	Compute target speed		E	S_T (knots)
11.	If additional observations are to be entered, read program card 1 and go to Step 6.			.

NOTE: ESTIMATION CRITERIA: A minimum of four bearing observations taken on a minimum of two own ship tracking legs must be entered prior to making estimates of target parameters.

User Instructions Flowchart



2. Sample Problems

a. Exact Bearing Information.

Own ship tracks on course 000° at 15 knots for 10 minutes and turns to 057° at 15 knots. Three bearing observations are made on the first leg, with the fourth observation made on the second leg. Exact bearing information is assumed available. The contact is tracked as follows:

<u>Time</u>	<u>Bearing</u>
1000	150.0
1005	148.2
1010	146.9
1020	154.7

Estimate the target's range, course and speed at time 1020.
(Answers: 12,920 yds, 045° , and 10.0 knots.)

The following additional observations are made:

<u>Time</u>	<u>Bearing</u>
1025	158.8
1030	163.0
1035	167.2

Estimate the target's range, course and speed at time 1035.
(Answers: 12540 yds, 045° , 10.0 knots.)

CARD 1 card	GSBE	Initialize
	000.00 ENT 15.00 GSEC	Own ship's course and speed
	10.00 ENT 150.00 GSEC	Observation #1 time and bearing
	10.05 ENT 148.20 GSEC	Observation #2 time and bearing
	10.10 ENT 146.90 GSEC	Observation #3 time and bearing
	057.00 ENT 15.00 GSEC	New own ship course and speed
	10.20 ENT 154.70 GSEC	Observation #4 time and bearing
	GSBE	Start estimation
CARD 2	GSBE 10.20 *** 12901.01 ***	Continue estimation Time of estimate Estimated target range in yards
	GSBE 48.37 ***	Estimated target course in degrees
	GSBE 9.86 ***	Estimated target speed in knots
CARD 1	10.25 ENT 158.80 GSEC	Observation #5 time and bearing
	10.30 ENT 163.00 GSEC	Observation #6 time and bearing
	10.35 ENT 167.20 GSEC	Observation #7 time and bearing
	GSBE	Start estimation
CARD 2	GSBE 10.35 *** 12540.49 ***	Continue estimation Time of revised estimate Revised estimated target range
	GSBE 46.22 ***	Revised estimated target course
	GSBE 9.95 ***	Revised estimated target speed

b. Inaccurate Bearing Information

Own ship tracks on course 000° at 15 knots for 10 minutes and turns to 056° at 15 knots. Three bearing observations are made on the first leg, with the fourth observation made on the second leg. The true target bearings have been altered by normal random variable with mean zero and variance 0.5 degrees squared. The contact is tracked as follows:

<u>Time</u>	<u>Bearing</u>
1000	150.0
1005	148.7
1010	146.0
1020	155.2

Estimate the target's range, course and speed at time 1020.
(Answers: 12,920 yds, 045° , and 10.0 knots.)

The following additional observations are made:

<u>Time</u>	<u>Bearing</u>
1025	159.7
1030	163.0
1035	166.4

Estimate the target's range, course and speed at time 1035.
(Answers: 12,740 yds, 045° , and 10.0 knots.)

<p>CARD 1</p> <p>000.00 ENT1 15.00 GSEa</p> <p>10.00 ENT1 150.00 GSEC</p> <p>10.05 ENT1 148.70 GSEC</p> <p>10.10 ENT1 146.00 GSEC</p> <p>056.00 ENT1 15.00 GSEa</p> <p>10.20 ENT1 155.20 GSEC</p> <p>GSEB</p>	<p>Initialize</p> <p>Own ship's course and speed</p> <p>Observation #1 time and bearing</p> <p>Observation #2 time and bearing</p> <p>Observation #3 time and bearing</p> <p>New own ship course and speed</p> <p>Observation #4 time and bearing</p> <p>Start estimation</p>
<p>CARD 2</p> <p>10.20 *** 5757.14 ***</p> <p>GSEB</p> <p>338.26 ***</p> <p>GSEB</p> <p>83.67 ***</p>	<p>Continue estimation</p> <p>Time of estimate</p> <p>Estimated target range in yards</p> <p>Estimated target course in degrees</p> <p>Estimated target speed in knots</p>
<p>CARD 1</p> <p>10.25 ENT1 159.70 GSEC</p> <p>10.30 ENT1 163.00 GSEC</p> <p>10.35 ENT1 166.40 GSEC</p> <p>GSEB</p>	<p>Observation #5 time and bearing</p> <p>Observation #6 time and bearing</p> <p>Observation #7 time and bearing</p> <p>Start estimation</p>
<p>CARD 2</p> <p>10.35 *** 12945.90 ***</p> <p>GSEB</p> <p>68.64 ***</p> <p>GSEB</p> <p>9.87 ***</p>	<p>Continue estimation</p> <p>Time of revised estimate</p> <p>Revised estimated target range</p> <p>Revised estimated target course</p> <p>Revised estimated target speed</p>

3. Program Storage Allocation and Listing

Registers

R0: t_1	S0: PS1; PS3; PS10
R1: $\sum W_i - \sum Z_i \tan B_i$	S1: $\sum \Delta t_i \tan^2 B_i$
R2: $-\sum W_i \tan B_i + \sum Z_i \tan^2 B_i$	S2: $\sum \Delta t_i^2 \tan^2 B_i$
R3: $\sum W_i \Delta t_i - \sum Z_i \Delta t_i \tan B_i$	S3: $-\sum \Delta t_i^2 \tan B_i$
R4: $-\sum W_i \Delta t_i \tan B_i + \sum Z_i \Delta t_i \tan^2 B_i$	S4: $\sum \Delta t_i$
R5: t_{i-1}	S5: $\sum \Delta t_i^2$
R6: t_i	S6: $\sum \tan B_i$
R7: W_i	S7: $\sum \tan^2 B_i$
R8: Z_i	S8: $\sum \Delta t_i \tan B_i$
R9: PS11; PS12; \hat{v} ; Δt_i	S9: $i = N$
RA: \dot{W}	
RB: \dot{Z}	
RC: Δt_i ; PS4; \hat{u}	
RD: $\tan B_i$; PS7; \hat{v}	
RE: $Z_i \tan B_i$; PS13; \hat{X}_1	
RI: i ; PS2; PS5; PS6; PS8; PS9; \hat{Y}_1	

NOTE: All summations are over the range of $i = 1, \dots, N$.

PS denotes 'partial-sum'.

Initial Flag Status and Use:

0: OFF, Unused	2: OFF, Unused
1: OFF, Used	3: OFF, Used

User Control Keys; Card 1:

A:	a: $C_0 \uparrow S_0 \rightarrow$
B:	b:
C: $t_i \uparrow B_i \rightarrow$	c:
D:	d:
E: Start \rightarrow	e: Initialize \rightarrow

User Control Keys; Card 2:

A: $\rightarrow t_i; \hat{R}_{T_i}$	a:
B:	b:
C: $\rightarrow \hat{C}_T$	c:
D:	d:
E: $\rightarrow \hat{S}_{T_i}$	e:

Card 1

001	*LBL1	21 01	Stores initial time, t_1 , for further use
002	ST00	35 02	
003	ST05	35 05	
004	RTN	24	
005	*LBL6	21 16 15	Initialization
006	CLRG	16-53	
007	P2S	16-51	
008	CLRG	16-53	
009	CLX	-51	
010	SF2	16 21 02	
011	RTN	24	
012	*LBL6	21 16 11	Calculates and stores own ship velocity components
013	-R	44	
014	ST08	35 12	
015	R4	-31	
016	ST0A	35 11	
017	RTN	24	
018	*LBLC	21 12	Input observation and bearing
019	TAN	43	
020	ST0D	35 14	Fills the matrix values
021	X2Y	-41	
022	HMS+	16 36	
023	ST06	35 06	
024	F22	16 23 02	
025	G8B1	23 01	Branches to store initial time
026	RCL0	36 00	
027	-	-45	
028	ST0C	35 13	
029	Z+	56	
030	ST0I	35 46	
031	R4	-31	
032	RCLC	36 13	
033	X2	53	
034	X	-35	
035	P2S	16-51	
036	ST-3	35-45 02	
037	RCLD	36 14	
038	X	-35	
039	ST+2	35-55 02	
040	RCLC	36 13	
041	RCLD	36 14	
042	X2	53	
043	X	-35	
044	ST+1	35-55 01	
045	P2S	16-51	
046	RCL6	36 06	
047	RCL5	36 05	
048	-	-45	
049	ENT1	-21	
050	ENT1	-21	

Card 1

051	FCLA	35 11
052	^	-35
053	ST+7	35-55 07
054	R4	-31
055	RCLB	36 12
056	x	-35
057	ST+8	35-55 08
058	RCL6	36 06
059	ST05	35 05
060	RCL7	36 07
061	ST+1	35-55 01
062	RCLD	36 14
063	x	-35
064	ST-2	35-45 02
065	RCLC	36 13
066	x	-35
067	ST-4	35-45 04
068	RCL7	36 07
069	RCLC	36 13
070	x	-35
071	ST+3	35-55 03
072	RCL8	36 08
073	RCLD	36 14
074	^	-35
075	ST0E	35 15
076	ST-1	35-45 01
077	RCLD	36 14
078	x	-35
079	ST+2	35-55 02
080	RCLC	36 13
081	x	-35
082	ST+4	35-55 04
083	RCLC	36 13
084	RCLC	36 13
085	x	-35
086	ST-3	35-45 03
087	RCLI	36 46
088	RTN	24
089	*LELE	21 15
090	F2S	16-51
091	1	01
092	CHS	-22
093	STX6	35-35 06
094	STX8	35-35 08
095	RCL8	36 08
096	RCL7	36 07
097	x	-35
098	RCL6	36 06
099	RCL1	36 01
100	x	-35

Start estimate calculations

Changes the sign of two matrix elements

(Partial sums stored throughout; PS)

Card 1

101	-	-45
102	RCL4	36 04
103	X ²	53
104	RCL9	36 09
105	RCL5	36 05
106	X	-35
107	-	-45
108	X	-35
109	ST00	35 00
110	RCL6	36 06
111	RCL4	36 04
112	X	-35
113	RCL9	36 09
114	RCL8	36 08
115	X	-35
116	-	-45
117	RCL8	36 08
118	X ²	53
119	RCL6	36 06
120	RCL3	36 03
121	X	-35
122	-	-45
123	X	-35
124	ST-0	35-45 00
125	RCL6	36 06
126	X ²	53
127	RCL9	36 09
128	RCL7	36 07
129	X	-35
130	-	-45
131	RCL8	36 08
132	P2S	16-51
133	RCL3	36 03
134	X	-35
135	RCL4	36 04
136	P2S	16-51
137	RCL4	36 04
138	X	-35
139	-	-45
140	X	-35
141	ST01	35 46
142	RCL8	36 08
143	X ²	53
144	RCL4	36 04
145	RCL1	36 01
146	X	-35
147	-	-45
148	RCL6	36 06
149	P2S	16-51
150	RCL1	36 01

Card 1

151	X	-35
152	RCL2	36 02
153	P2S	16-51
154	RCL9	36 05
155	X	-35
156	-	-45
157	X	-35
158	RCL1	36 4E
159	-	-45
160	ST00	35 13
161	RCL0	36 00
162	X	-35
163	P2S	16-51
164	ST03	35 05
165	P2S	16-51
166	RCL6	36 06
167	X2	53
168	RCL9	36 09
169	RCL7	36 07
170	X	-35
171	-	-45
172	RCL6	36 0E
173	RCL5	36 05
174	X	-35
175	RCL4	36 04
176	RCL3	36 03
177	X	-35
178	-	-45
179	X	-35
180	ST01	35 4E
181	RCL8	36 08
182	X2	53
183	RCL4	36 04
184	RCL1	36 01
185	X	-35
186	-	-45
187	RCL6	36 0E
188	RCL4	36 04
189	X	-35
190	RCL9	36 05
191	RCL8	36 08
192	X	-35
193	-	-45
194	X	-35
195	RCL1	36 4E
196	-	-45
197	ST00	35 14
198	RCL6	36 06
199	RCL4	36 04
200	X	-35

Card 1

201	RCL9	36 05
202	RCL8	36 08
203	X	-35
204	-	-45
205	RCL8	36 08
206	P/S	16-51
207	RCL2	36 02
208	X	-35
209	RCL4	36 04
210	P/S	16-51
211	RCL6	36 06
212	X	-35
213	-	-45
214	X	-35
215	STOI	35 46
216	RCL8	36 08
217	RCL7	36 07
218	X	-35
219	RCL6	36 06
220	RCL1	36 01
221	X	-35
222	-	-45
223	R/S	51

Card 2

001	*LEL4	31 11	Calculation of estimates
002	RCL4	36 04	
003	F28	16-51	
004	RCL1	36 01	
005	X	-35	
006	RCL3	36 03	
007	F28	16-51	
008	RCL9	36 05	
009	X	-35	
010	-	-45	
011	X	-35	
012	RCL1	36 46	
013	-	-45	
014	RCL0	36 14	
015	X	-35	
016	F28	16-51	
017	RCL9	36 05	
018	X2Y	-41	
019	-	-45	
020	ST09	35 05	
021	F28	16-51	
022	RCL6	36 06	
023	X2	53	
024	RCL9	36 05	
025	RCL7	36 07	
026	X	-35	
027	-	-45	
028	RCL8	36 08	
029	RCL3	36 03	
030	X	-35	
031	RCL4	36 04	
032	RCL2	36 02	
033	X	-35	
034	-	-45	
035	X	-35	
036	ST01	35 46	
037	RCL8	36 08	
038	X2	53	
039	RCL4	36 04	
040	RCL1	36 01	
041	X	-35	
042	-	-45	
043	RCL6	36 06	
044	RCL8	36 08	
045	X	-35	
046	RCL9	36 09	
047	RCL1	36 01	
048	X	-35	
049	-	-45	
050	X	-35	

051	RCL1	36 46	
052	-	-45	
053	STOE	35 15	
054	RCL0	36 00	
055	X	-35	
056	ST00	35 00	
057	RCL6	36 06	
058	RCL4	36 04	
059	X	-35	
060	RCL9	36 09	
061	RCL8	36 08	
062	X	-35	
063	-	-45	
064	RCL8	36 08	
065	RCL1	36 01	
066	X	-35	
067	PCL6	36 06	
068	PCL2	36 02	
069	X	-35	
070	-	-45	
071	X	-35	
072	ST01	35 46	
073	PCL8	36 08	
074	RCL7	36 07	
075	X	-35	
076	RCL6	36 06	
077	RCL1	36 01	
078	X	-35	
079	-	-45	
080	RCL4	36 04	
081	RCL8	36 08	
082	X	-35	
083	RCL9	36 09	
084	RCL3	36 03	
085	X	-35	
086	-	-45	
087	X	-35	
088	RCL1	36 46	
089	-	-45	
090	RCL0	36 14	
091	X	-35	
092	RCL0	36 00	
093	X \neq Y	-41	
094	-	-45	
095	P \neq S	16-51	
096	RCL9	36 09	
097	X \neq Y	-41	
098	=	-24	
099	ST09	35 09	^
100	RCL5	36 15	v

Card 2

101	X	-35	\hat{u}
102	RCLC	36 13	
103	XZY	-41	
104	-	-45	
105	RCLD	36 14	
106	÷	-24	
107	STOC	35 13	
108	PCL5	36 09	
109	STOD	35 14	
110	PZS	16-51	
111	RCL6	36 06	
112	RCL8	36 06	
113	X	-35	
114	PCL9	36 09	
115	PCL1	36 01	\hat{y}_1
116	÷	-35	
117	-	-45	
118	X	-35	
119	CH5	-22	
120	RCL6	36 06	
121	RCL4	36 04	
122	X	-35	
123	RCL9	36 09	
124	RCL8	36 06	
125	X	-35	
126	-	-45	
127	RCLC	36 13	
128	X	-35	
129	-	-45	
130	RCL6	36 06	\hat{y}_1
131	PZS	16-51	
132	PCL1	36 01	
133	X	-35	
134	RCL2	36 02	
135	PZS	16-51	
136	RCL9	36 09	
137	X	-35	
138	-	-45	
139	+	-55	
140	RCL6	36 06	
141	XZ	53	
142	RCL9	36 09	
143	RCL7	36 07	
144	X	-35	
145	-	-45	
146	÷	-24	
147	STOI	35 46	
148	PZS	16-51	
149	PCL1	36 01	
150	PZS	16-51	

Card 2

151	RCL8	36 08	
152	RCLD	36 14	
153	x	-35	
154	-	-45	
155	RCL4	36 04	
156	RCLC	36 13	
157	x	-35	
158	-	-45	
159	RCLF	36 06	
160	RCLI	36 46	
161	x	-35	
162	-	-45	
163	RCL9	36 05	\hat{X}_1
164	÷	-24	
165	STOE	35 15	
166	1	01	Change sign of two matrix elements
167	CHS	-22	
168	ST×6	35-35 05	
169	ST×8	35-35 08	
170	P÷S	16-51	
171	RCL6	36 06	t_i
172	→HMS	16 35	
173	FRTX	-14	
174	RCL6	36 06	
175	RCL0	36 00	
176	-	-45	
177	ST09	35 05	
178	RCLC	36 13	
179	x	-35	
180	RCLF	36 15	
181	+	-55	
182	RCL7	36 07	
183	-	-45	
184	X²	53	
185	RCL9	36 05	
186	RCLD	36 14	
187	x	-35	
188	RCLI	36 46	
189	+	-55	
190	RCL8	36 08	
191	-	-45	
192	X²	53	
193	+	-55	
194	√X	54	$\hat{R}_T = ((X_1 + u, t_i - W_i)^2 + (Y_1 + v, t_i - Z_i)^2)^{1/2}$
195	2	02	
196	EEV	-23	(in nautical miles)
197	3	03	
198	x	-35	\hat{R}_T in yards
199	FRTX	-14	
200	SF1	16 21 01	

Card 2

201	RTN	24	
202	*LBLD	21 13	
203	PCLD	36 13	
204	PCLD	36 14	
205	+F	34	
206	XZY	-41	
207	XGP	16-45	
208	GEB2	23 02	
209	PRTX	-14	\hat{C}_T
210	CF1	16 22 01	
211	FTN	24	
212	*LBL2	21 02	
213	3	03	
214	6	06	
215	0	00	
216	+	-55	
217	RTN	24	
218	*LBLE	21 15	
219	F12	16 23 01	
220	GT09	22 09	
221	R4	-31	\hat{S}_T
222	PRTX	-14	
223	SPC	16-11	
224	RTN	24	

E. Mathematical Analysis

a. Assumptions

All calculations use a rectangular coordinate system as defined below. Additionally it is assumed the following quantities are accurately known (although the accuracy of the observed bearing varies):

- (1) time
- (2) target bearings
- (3) observer course and speed
- (4) observer initial position.

b. Symbology

(1) Observer

- (a) W: East-West position
- (b) Z: North-South position
- (c) (W_i, Z_i) : position at ith observation.

(2) Target

- (a) X: East-West position
- (b) Y: North-South position
- (c) (X_i, Y_i) : position at ith observation
- (d) u: East-West velocity component
- (e) v: North-South velocity component.

(3) Other

- (a) B_i : measured bearing from observer to target at the ith observation.
- (b) t_i : time of the ith observation
- (c) Δt_i : elapsed time between ith and first observation.
- (d) $i = 1$ is the initial observation.

c. Development

The geometry for a two leg TMA is shown in Figure 1. The target motion/position at any time can be described in terms of its initial position (X_1, Y_1) , and its velocity components $(u$ and $v)$, which are unknown and the elapsed time, Δt_i , which is known by: $X_i = X_1 + u\Delta t_i$ and $Y_i = Y_1 + v\Delta t_i$. Knowledge of the target bearing leads to the following:

$$\tan B_i = \frac{X_i - W_i}{Y_i - Z_i} = \frac{X_1 + u\Delta t_i - W_i}{Y_1 + v\Delta t_i - Z_i}$$

or

$$X_1 - Y_1 \tan B_i + u\Delta t_i - v\Delta t_i \tan B_i = W_i - Z_i \tan B_i \quad (1)$$

Equation (1) has four known variables $(W_i, Z_i, \Delta t_i, B_i)$ and the four unknown target variables (X_1, Y_1, u, v) . Define estimates for the four unknowns as $\hat{X}_1, \hat{Y}_1, \hat{u}$, and \hat{v} and define an error, e_i , that represents the errors due to the use of estimates in Equation (1) at each observation:

$$e_i = \hat{X}_1 - \hat{Y}_1 \tan B_i + \hat{u}\Delta t_i - \hat{v}\Delta t_i \tan B_i - W_i + Z_i \tan B_i.$$

The least squares estimates of the variables X_1, Y_1, u and v are those values which minimize the expression

$$\sum_{i=1}^n e_i^2 = E(X_1, Y_1, u, v)$$

where n is the number of observations.

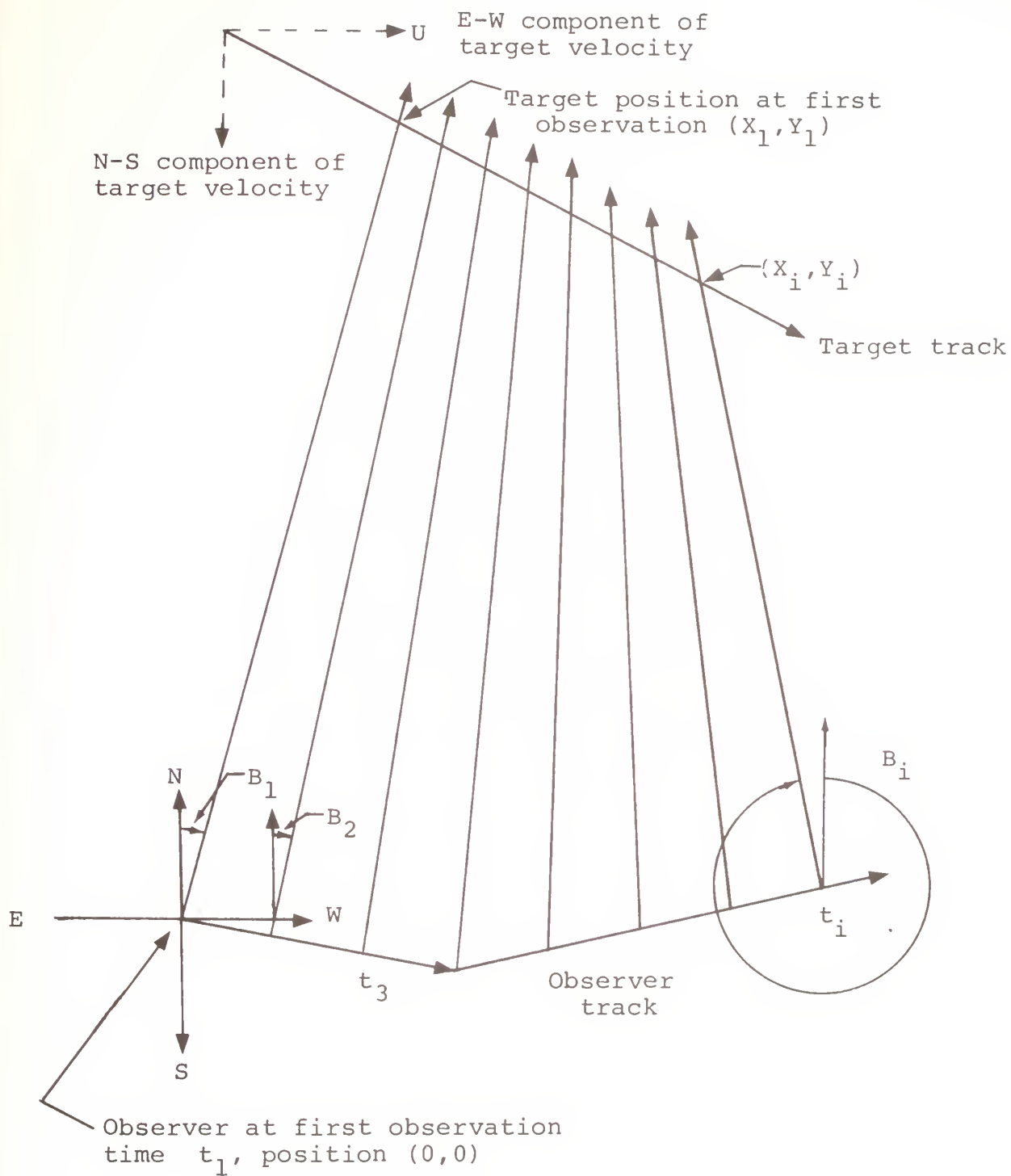


FIGURE 1. TMA Geometry

To minimize, take the partial derivatives of E with respect to each variable and set each partial derivative equal to zero. The resulting set of four linear equations in four unknowns (Figure 2) can be solved at each bearing observation if $i \geq 4$ and an observer course or speed change has occurred during the TMA period.

The following target parameters can then be calculated at each successive observation:

$$\text{Target Range} = \sqrt{(\hat{X}_i - W_i)^2 + (\hat{Y}_i - Z_i)^2}$$

$$\text{Target Speed} = \sqrt{\hat{u}^2 + \hat{v}^2}$$

Target Course = $\arctan(\hat{u}/\hat{v})$ with appropriate logic to select the correct coordinate quadrant.

When calculating these parameters appropriate constants are required to insure proper units.

F. Reference.

1. P. W. Marzluff and R. C. Pilcher, "Basic Calculator Methods of Bearings-Only Target Motives Analyses for a Moving Sensor (U). Naval Postgraduate School Thesis, December 1978.

FOUR BEARING TMA NORMAL EQUATIONS

$$\begin{bmatrix} n \\ - \sum_{i=1}^n \tan B_i \\ \sum_{i=1}^n \Delta t_i \tan B_i \\ - \sum_{i=1}^n \Delta t_i \tan^2 B_i \\ \sum_{i=1}^n \Delta t_i \tan B_i \\ - \sum_{i=1}^n \Delta t_i \tan^2 B_i \end{bmatrix} \begin{bmatrix} - \sum_{i=1}^n \Delta t_i \tan B_i \\ \sum_{i=1}^n \Delta t_i \tan^2 B_i \\ - \sum_{i=1}^n \Delta t_i^2 \tan B_i \\ \sum_{i=1}^n \Delta t_i^2 \tan^2 B_i \\ - \sum_{i=1}^n \Delta t_i^2 \tan B_i \\ \sum_{i=1}^n \Delta t_i^2 \tan^2 B_i \end{bmatrix} \begin{bmatrix} \hat{X}_l \\ \hat{Y}_l \\ \hat{u} \\ \hat{v} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^n W_i - \sum_{i=1}^n Z_i \tan B_i \\ - \sum_{i=1}^n W_i \tan B_i + \sum_{i=1}^n Z_i \tan^2 B_i \\ \sum_{i=1}^n W_i \Delta t_i - \sum_{i=1}^n Z_i \Delta t_i \tan B_i \\ - \sum_{i=1}^n W_i \Delta t_i \tan B_i + \sum_{i=1}^n Z_i \Delta t_i \tan^2 B_i \end{bmatrix}$$

FIGURE 2

VIII. NAVAL GUNFIRE SUPPORT GRID SPOT CONVERSIONS, TRUE WIND,
AND TIME OF FLIGHT/MAXIMUM ORDINATE COMPUTATIONS FOR
5-INCH/54 PROJECTILE by LT Keith P. Curtis

A. Problem Statement

The success of Naval Gunfire Support operations in the Combat Information Center (CIC) is a function of rapid information processing and relay. Specifically, substantial error can be introduced by inaccurate grid spot conversions and to a lesser degree by improper computations of true wind. Also, commencement of a fire mission can be delayed waiting for Time of Flight (TOF) and/or Maximum Ordinate (Max Ord) information.

The inherent error of rapid calculations can be minimized by the use of the handheld programmable calculator. This paper addresses the use of the Hewlett Packard HP-67 to perform grid-spot conversions; compute true wind, TOF, and Max Ord.

B. Operational Analysis

The objective of this program is to provide a one-card program to accommodate the following:

1. Correct for magnetic variance for a geographic area.
2. Accept the Observer Target Line (OTL) in either mils magnetic or degree magnetic.
3. Perform precise grid spot conversion.
4. Provide Time of Flight information.

5. Provide Maximum Ordinate information.
6. Compute true wind.

C. Computational Algorithm

1. Enter magnetic variance.
2. Enter OTL either in mils magnetic or degree magnetic.
3. Enter observer "spots": left-right, add-drop (in yards).
4. Convert spots to East-West, North-South.
5. Enter range of shot and compute TOF and max ord.
6. Enter own ship course and speed, and relative wind to compute true wind.
7. Repeat Steps 3 through 6 as necessary.

D. HP-67 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Read program card (both sides)			
2.	Enter magnetic variation (+ for East, - for West). If no variation is entered, 0 is used.	mag.var.	f b	
3.	Enter Observer Target Line in either:			
a.	degrees magnetic, or		A	OTL °T
b.	mils magnetic		f a	OTL °T
4 a.	Enter Left/Right spot (- for left, + for right)	L/R	↑	
b.	Enter Add/Drop spot (- for drop, + for add)	A/D	B	
c.	Display E/W spot (+ E, - W)			E/W spot
d.	Display N/S spot (+ N, - S)			N/S spot
e.	Optional: Recover the E/W spot		h x $\frac{1}{y}$ y	E/W spot
	Repeat Step 4 as required until OTL changes			
5.	Compute TOF and max ord			
a.	Enter target range, display:	range	C	
b.	Time of Flight			TOF
c.	Max Ord			Max ord
	Optimal: Recover TOF		h R ↑	
6.	Compute true wind.			
a.	Enter own ship's course and speed in the form SS.CCC where SS is the speed in integer knots and CCC is a three-digit course	SS.CCC	↑	
b.	Enter relative wind	SS.CCC	D	
c.	True wind is displayed			SS.CCC

Note: Use of the keys C or D do not require an OTL input.

2. Sample Problem

Own ship:	035T 12KTS
Relative wind:	225R 6KTS
Magnetic variance:	7E
OTL:	1930 mils mag
Spot:	Left 250, Drop 150
Range:	9600 yds

West 27, North 290

TOF: 16 seconds

Max Ord: 1050ft

True Wind: 230T 17KTS

7. GSEB
 1930. GSEB
 116. ***
 -250. ENT
 -150. GSEB
 -27. ***
 290. ***

 9600. GSEB
 16. ***
 1050. ***

 12.035 ENT
 6.225 GSEB
 17.230 ***

3. Program Storage Allocation and Listings

Registers

R0:	OTL °T	S0:		RA:	\log_{10} range
R1:	} TOF/ Max Ord Coeff	S1:		RB:	Ship's heading
R2:		S2:		RC:	L-R spot
R3:		S3:		RD:	A-D spot
R4:		S4:	ΣX	RE:	
R5:		S5:		RI:	Control
R6:		S6:	ΣY		
R7:		S7:			
R8:		S8:			
R9:	mag.var.	S9:			

Initial Flag Status and Use

0:	OFF, Unused	2:	OFF, Unused
1:	OFF, Unused	3:	OFF, Unused

User Control Keys

A:	OTL (degrees)	a:	OTL (mils)
B:	L-R spot, A-D spot	b:	mag.var.
C:	Range	c:	
D:	O/S c/S rel wind	d:	
E:		e:	

0.000000000	0
0.777500000	1
2.166000000	2
-0.207100000	3
0.261700000	4
0.072100000	5
0.109300000	6
0.850500000	7
0.283100000	8
0.000000000	9
0.000000000	A
0.000000000	B
0.000000000	C
0.000000000	D
0.000000000	E
0.000000000	I

COEFFICIENTS FOR MAX ORD/TOF EQNS

PRESTORED DATA

001	*LELB	21 16 12	STORE MAG VAR
002	DSP0	-63 00	
003	ST09	35 00	CONVERT MILS TO DEGREES
004	R/S	51	
005	*LELB	21 16 11	CONVERT QTL TO TRUE BRG
006	.	-62	
007	0	00	CONVERT OBSERVER GRID SPOTS TO E-W, N-S SPOTS
008	5	05	
009	6	06	CONVERT TRUE WIND
010	2	02	
011	5	05	CONVERT TRUE WIND
012	X	-35	
013	*LELB	21 11	CONVERT TRUE WIND
014	DSP0	-63 00	
015	RCL5	36 00	CONVERT TRUE WIND
016	+	-55	
017	X00	16-45	CONVERT TRUE WIND
018	GSEe	23 16 15	
019	ST00	35 00	CONVERT TRUE WIND
020	R/S	51	
021	*LELB	21 12	CONVERT TRUE WIND
022	DSP0	-63 00	
023	XZY	-41	CONVERT TRUE WIND
024	+P	34	
025	XZY	-41	CONVERT TRUE WIND
026	RCL0	36 00	
027	-	-45	CONVERT TRUE WIND
028	XZY	-41	
029	+P	44	CONVERT TRUE WIND
030	PRTX	-14	
031	XZY	-41	CONVERT TRUE WIND
032	R/S	51	

033	*LELB	21 12	CONVERT TRUE WIND
034	DSP0	-63 00	
035	EEX	-23	CONVERT TRUE WIND
036	3	03	
037	+	-24	CONVERT TRUE WIND
038	LOG	16 32	
039	ST04	35 11	CONVERT TRUE WIND
040	RCL6	36 06	
041	RCL7	36 07	CONVERT TRUE WIND
042	RCL8	36 08	
043	RCLA	36 11	CONVERT TRUE WIND
044	X	-35	
045	+	-55	CONVERT TRUE WIND
046	RCLA	36 11	
047	X	-35	CONVERT TRUE WIND
048	+	-55	
049	10X	16 32	CONVERT TRUE WIND
050	PRTX	-14	
051	4	04	CONVERT TRUE WIND
052	ST01	35 46	
053	RCL5	36 05	CONVERT TRUE WIND
054	*LELB	21 16 12	
055	RCLA	36 11	CONVERT TRUE WIND
056	X	-35	
057	RCL1	36 45	CONVERT TRUE WIND
058	+	-55	
059	DSP1	16 25 46	CONVERT TRUE WIND
060	GT00	22 16 12	
061	10X	16 32	CONVERT TRUE WIND
062	R/S	51	
063	*LELB	21 14	CONVERT TRUE WIND
064	DSP0	-63 00	
065	P2S	16-51	CONVERT TRUE WIND
066	CLRG	16-53	
067	P2S	16-51	CONVERT TRUE WIND
068	XZY	-41	
069	GEBd	23 16 14	CONVERT TRUE WIND
070	S-	16 56	
071	R1	16-31	CONVERT TRUE WIND
072	RCLB	36 12	
073	+	-55	CONVERT TRUE WIND
074	GSEd	23 16 14	
075	S+	56	CONVERT TRUE WIND
076	RCL2	36 56	
077	+P	34	CONVERT TRUE WIND
078	RND	16 24	
079	INT	16 34	CONVERT TRUE WIND
080	XZY	-41	
081	X00	16-45	CONVERT TRUE WIND
082	GSEe	23 16 15	
083	EEX	-23	CONVERT TRUE WIND
084	3	03	
085	+	-24	CONVERT TRUE WIND
086	+	-55	
087	DSP3	-63 03	CONVERT TRUE WIND
088	R/S	51	

089	*LBLd	21 16 14
090	INT	16 34
091	LSTX	16-63
092	FRC	16 44
093	STOB	35 12
094	EEX	-23
095	3	03
096	x	-35
097	X*Y	-41
098	+R	44
099	RTN	24
100	*LBLe	21 16 15
101	3	03
102	6	0c
103	0	00
104	+	-55
105	RTN	24
106	R/S	51

E. Geometric/Mathematical Analysis

1. Grid-Spot Conversion

The conversion of grid spots oriented to an Observer Target Line to an East-West, North-South orientation for input to a shipboard GFCS can be accomplished by a rotation of the OTL counterclockwise to 000 degree True after the OTL has been corrected to true bearing (Figure 1).

The rotation formulas are

$$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} ,$$

or

$$x' = x \cos \theta + y \sin \theta \quad \text{and} \quad y' = -x \sin \theta + y \cos \theta ,$$

where

$$\theta = \text{OTL } ^\circ\text{T}$$

$$x = \text{L/R Spot,}$$

$$y = \text{A/D Spot,}$$

$$x' = \text{E-W Spot,}$$

$$y' = \text{N-S Spot}$$

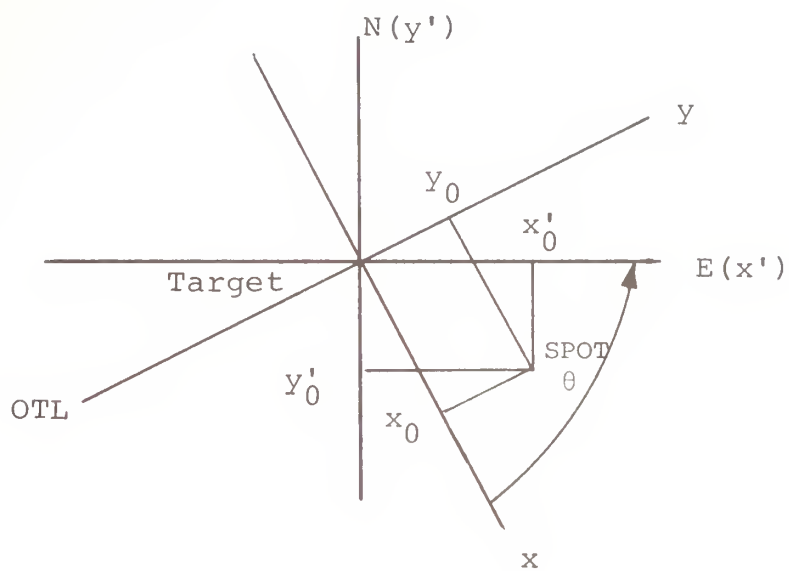


FIGURE 1: Rotation Geometry

2. TOF and Maximum Ordinate

This information is tabulated for the 5"/54 Projectile in BuOrd Publication OP1182 (Range Table for 5"/54). To find an equation which would best fit the tabulated data, a log transformation was made followed by a parabolic curve fit.

The time of Flight Equation is

$$f(x) = 10^{(0.1083 + 0.8505 \log x + 0.2831(\log x)^2)}$$

This equation generates solutions within 1 second of the tabulated values for ranges under 22,000 yds. The same approach was made in determining an equation for maximum ordinate.

The Maximum Ordinate Equation is

$$f(x) = 10^{(0.7775 + 2.1666 \log x - 0.2071(\log x)^2 + 0.2617(\log x)^3 + 0.0721(\log x)^4)}$$

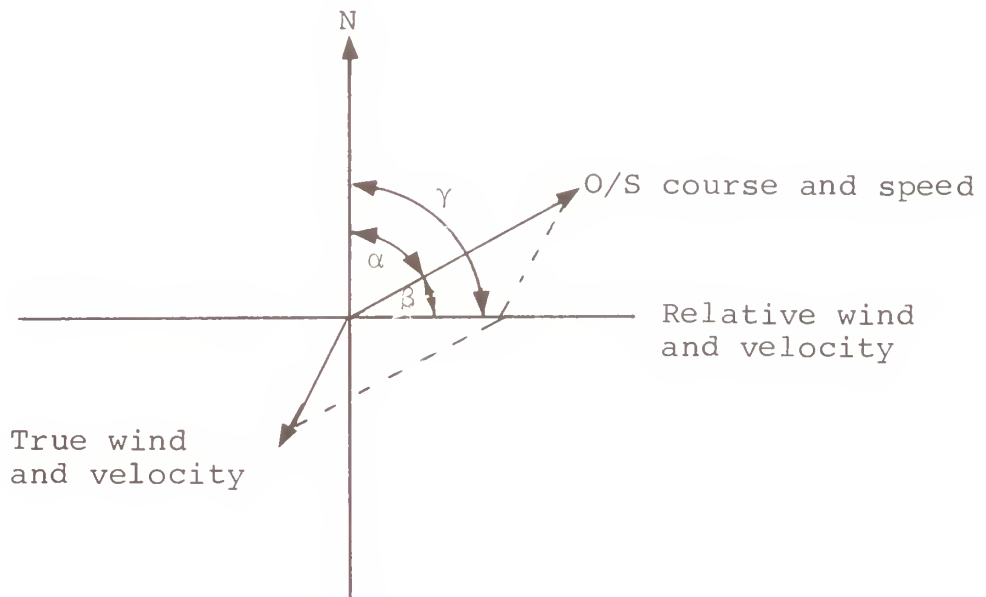
This equation generates solutions within 55ft of the tabulated values for ranges less than 19,000 yds. (Within 10ft for range less than 13,000 yds, and with 1 ft for ranges less than 7000 yds.)

The algorithm uses a nested polynomial to preserve accuracy in the calculation of the exponent. For example, Time of Flight is computed as follows:

$$f(x) = 10^{((0.2831 \log x + 0.8505) \log x + 0.1083)}$$

3. True Wind.

This computation is simple vector arithmetic. Relative wind is converted to Apparent wind by adding the ship's heading to the relative bearing, then converted to rectangular coordinates. Own ship's course and speed are then converted to rectangular coordinates and subtracted from apparent wind vector. The result is true wind which is converted to polar coordinates (Figure 2).



α = ship's heading

β = relative wind bearing

γ = apparent wind bearing

FIGURE 2. Wind vectors

IX. NORMAL MODE THEORY by LT J. M. Stone

A. Problem Statement

This program determines the number of normal modes that will propagate in a given ocean model. The ocean model must have either a rigid bottom or pressure release bottom. Also provided with each mode is the cutoff frequency (f_c), group velocity (C_g), and phase velocity (C_p). The user provides the speed of sound in water in m/sec (C_0), water depth in m(d), and frequency of the source in Hz(f).

B. Operational Analysis

None.

C. Computational Algorithm

Not submitted.

D. HP-67/07 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Read program card			
2.	Initialize		f e	1.00
3.a.	Sound velocity (m/sec)	C ₀	↑	
b.	Water depth (m)	d	↑	
c.	Source frequency (Hz)	f		
4.	Bottom type: Either			
a.	Rigid Bottom, or	none	A	
b.	Pressure release bottom		B	
5.	Output sequence			
a.	Mode number			n
b.	Cutoff frequency for mode n			f _C
c.	Group velocity for mode n			C _g
d.	Phase velocity for mode n			C _p
e.	Display mode number			n
6.	Continue from Step 4a or 4b depending upon original bottom type. This process continues until the highest mode that will propagate for the given conditions has been displayed when [A] or [B] is pressed and the next mode will not propagate then the program displays the mode number of the last mode that will propagate.			

2. Sample Problems

Example 1.

$$C_0 = 1500 \text{ m/sec}$$

Rigid bottom

$$d = 15 \text{ m}$$

$$f = 150 \text{ Hz}$$

1500.00	6SEe	Initializes Program
15.00	ENT:	
150.00	6SEe	
1.00	***	Flashes mode number (n)
25.00	***	Flashes f_C for $n = 1$
1479.02	***	Flashes C_g for $n = 1$
1521.28	***	Flashes C_p for $n = 1$ (Stops, displaying mode no.)
	6SBA	
2.00	***	Flashes n
75.00	***	Flashes f_C for $n = 2$
1299.04	***	Flashes C_g for $n = 2$
1732.05	***	Flashes C_p for $n = 2$ (Stops, displaying mode no.)
	6SBA	
3.00	***	Flashes n
125.00	***	Flashes f_C for $n = 3$
829.16	***	Flashes C_g for $n = 3$
2713.60	***	Flashes C_p for $n = 3$ (Stops, displaying mode no.)
	6SBA	
3.00	***	Mode 4 will not propagate under
	6SBA	these conditions so regardless of how many times A
3.00	***	is pressed, mode 3 is displayed.

Example 2:

$$C_0 = 1500 \text{ m/sec}$$

Pressure Release Bottom

$$d = 15 \text{ m}$$

$$f = 150 \text{ Hz}$$

<p> GSEe 1500.00 ENT: 15.00 ENT1 150.00 GSEe 1.00 *** 50.00 *** 1414.21 *** 1590.95 *** GSEe 2.00 *** 100.00 *** 1118.03 *** 2012.46 *** GSEe 2.00 *** GSEe 2.00 *** </p>	<p>Initializes Program</p> <p>Flashes mode no.</p> <p>Flashes f_C for mode 1</p> <p>Flashes C_g for mode 1</p> <p>Flashes C_p for mode 1 (Stops, displaying mode no.)</p> <p>Flashes mode no.</p> <p>Flashes f_C for mode 2</p> <p>Flashes C_g for mode 2</p> <p>Flashes C_p for mode 2 (Stops, displaying mode no.)</p> <p>Only two modes will propagate</p>
---	---

NOTE: If such conditions exist such that no modes will propagate, 0.00 is displayed.

The program stops between modes and requires the user to initiate the next mode in order to allow the user sufficient time to write down the information presented.

3. Program Storage Allocations and Program Listing

Registers:

R0: f	S0:	RA:
R1: d	S1:	RB:
R2: C_0	S2:	RC:
R3: n	S3:	RD:
R4: f_C	S4:	RE:
R5: $\sqrt{1 - (f_C/f)^2}$	S5:	RI:
R6:	S6:	
R7:	S7:	
R8:	S8:	
R9:	S9:	

Initial Flag Status and Uses

0: OFF, Unused	2. OFF, Unused
1: OFF, Unused	3: OFF, Unused

User Control Keys

A: Rigid Bottom	a:
B: Pressure Release Bottom	b:
C:	c:
D:	d:
E:	e:

001	*LBL	21 16 15	Initializes Program
002	1	01	
003	STO3	35 03	n = 1
004	SF2	16 21 02	Controls storage of inputs on first pass
005	R/S	51	Input parameters at this stop
006	*LBLA	21 11	Case I--Rigid Bottom
007	F2?	16 23 02	
008	GSB1	23 01	Stores inputs on first pass
009	RCL3	36 03	
010	.	-62	
011	5	05	
012	x	-35	
013	.	-62	
014	2	02	Calculates f_C
015	5	05	
016	-	-45	
017	RCL2	36 02	$f_C = \frac{C_0(2n-1)}{4d}$
018	x	-35	
019	RCL1	36 01	
020	=	-24	
021	RCL0	36 00	
022	XZY	-41	
023	X*Y?	16-32	Is $f_C \geq f$
024	X>Y?	16-34	
025	GT04	22 04	Yes--display last mode # and stop
026	ST04	35 04	No--continue
027	RCL3	36 03	
028	PFTX	-14	Flash new mode #
029	RCL4	36 04	
030	PRTX	-14	Flash f_C
031	GSB2	23 02	Computes C_g and C_p
032	*LBL3	21 03	Increments mode no. for next iteration
033	1	01	
034	ST+3	35-55 03	
035	RCL3	36 03	
036	1	01	
037	-	-45	
038	R/S	51	Stops, displays mode # of run just completed
039	*LBLB	21 12	Case II--Pressure Release Bottom
040	F2?	16 23 02	
041	GSB1	23 01	Stores inputs on first pass
042	RCL3	36 03	
043	RCL2	36 02	
044	x	-35	Calculates f_C
045	RCL1	36 01	
046	=	-24	
047	2	02	
048	=	-24	$f_C = \frac{C_0(2n)}{4d}$
049	RCL0	36 00	

050	XZY	-41
051	XZY?	16-32
052	XZY?	16-34
053	GT04	22 04
054	ST04	35 04
055	RCL3	36 03
056	FPTX	-14
057	FCL4	36 04
058	FPTX	-14
059	GSB2	23 02
060	GT03	22 03
061	*LBL1	21 01
062	ST00	35 00
063	R↓	-31
064	ST01	35 01
065	R↓	-31
066	ST02	35 02
067	RTN	24
068	*LBL2	21 02
069	RCL4	36 04
070	RCL0	36 00
071	÷	-24
072	X²	53
073	CHS	-22
074	1	01
075	+	-55
076	JX	54
077	ST05	35 05
078	RCL2	36 02
079	x	-35
080	FPTX	-14
081	RCL2	36 02
082	RCL5	36 05
083	÷	-24
084	FPTX	-14
085	RTN	24
086	*LBL4	21 04
087	RCL3	36 03
088	1	01
089	-	-45
090	R/S	51

Is $f_C \geq f$

Display last mode no.

Yes--and stop

No--continue

Flash new mode no.

Flash f_C

Computes C_g and C_p increments mode # for next iteration

Subroutine 1--Stores inputs on first pass only

Subroutine 2--Calculates C_g and C_p

$$C_g = C_0 \sqrt{1 - (f_C/f)^2}$$

$$C_p = C_0 / \sqrt{1 - (f_C/f)^2}$$

Flash C_g

Flash C_p

Allows the last mode number to be displayed after each iteration

E. Mathematical Analysis

Consider a shallow water ocean model with depth d . According to Normal Mode Theory the sound pressure at any point can be determined by the solution of the wave equation and is given by

$$p_n = A_n \sin K_{nz} z \exp[i(\omega t - K_{nz} x)]$$

Sound will propagate at different angles and this is what gives rise to the various modes. As θ , in Figure 1 approaches zero, the sound will not propagate because it is merely bouncing up and down off the surface and bottom (no x-direction of travel). This determines the cutoff frequency for mode n .

The cutoff frequency is dependent upon the boundary conditions at the surface and bottom because the solution of the wave equation is dependent upon the boundary conditions. The surface is always considered to be a pressure release boundary. As the mode number increases the cutoff frequency for that mode is higher also. When the cutoff frequency exceeds the frequency of the source then that mode will not propagate, nor will higher modes.

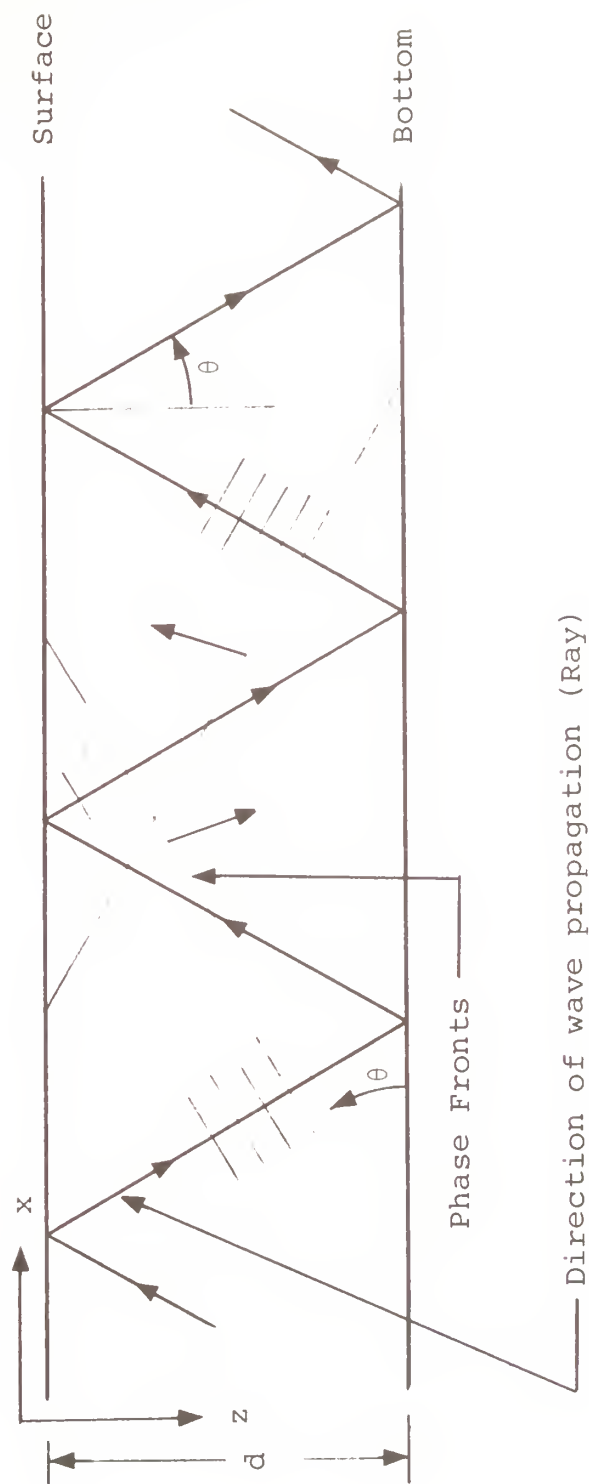


FIGURE 1. Wave Propagation Geometry

Case I: Rigid Bottom

For the rigid bottom, the boundary condition yields a

$$K_{nz} = \frac{(2n-1)\pi}{2d}, \quad n = 1, 2, 3 \quad (\text{mode } \#)$$

and

$$f_C = \frac{C_0}{2} \frac{(2n-1)\pi}{2d}.$$

Case II: Pressure Release Bottom

For the pressure release bottom, the boundary condition yields

$$K_{nz} = \frac{n\pi}{d}, \quad n = 1, 2, 3, \dots \quad (\text{mode } \#)$$

and

$$f_C = \frac{C_0}{2\pi} \frac{n\pi}{d}.$$

The remaining values calculated are the group velocity $C_g = C_0 \cos \phi$ and the phase velocity $C_p = C_0 / \cos \phi$, where $\cos \phi = [1 - (f_C/f)^2]^{1/2}$, f_C is the cutoff frequency, f is the source frequency, and C_0 is the velocity of sound. The geometry of these quantities is shown in Figure 2.

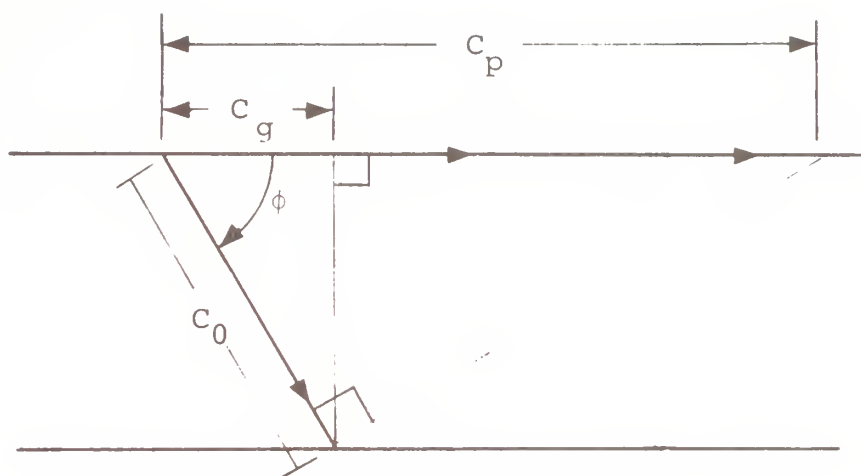
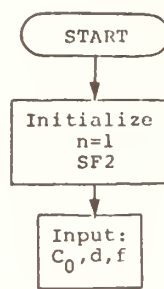
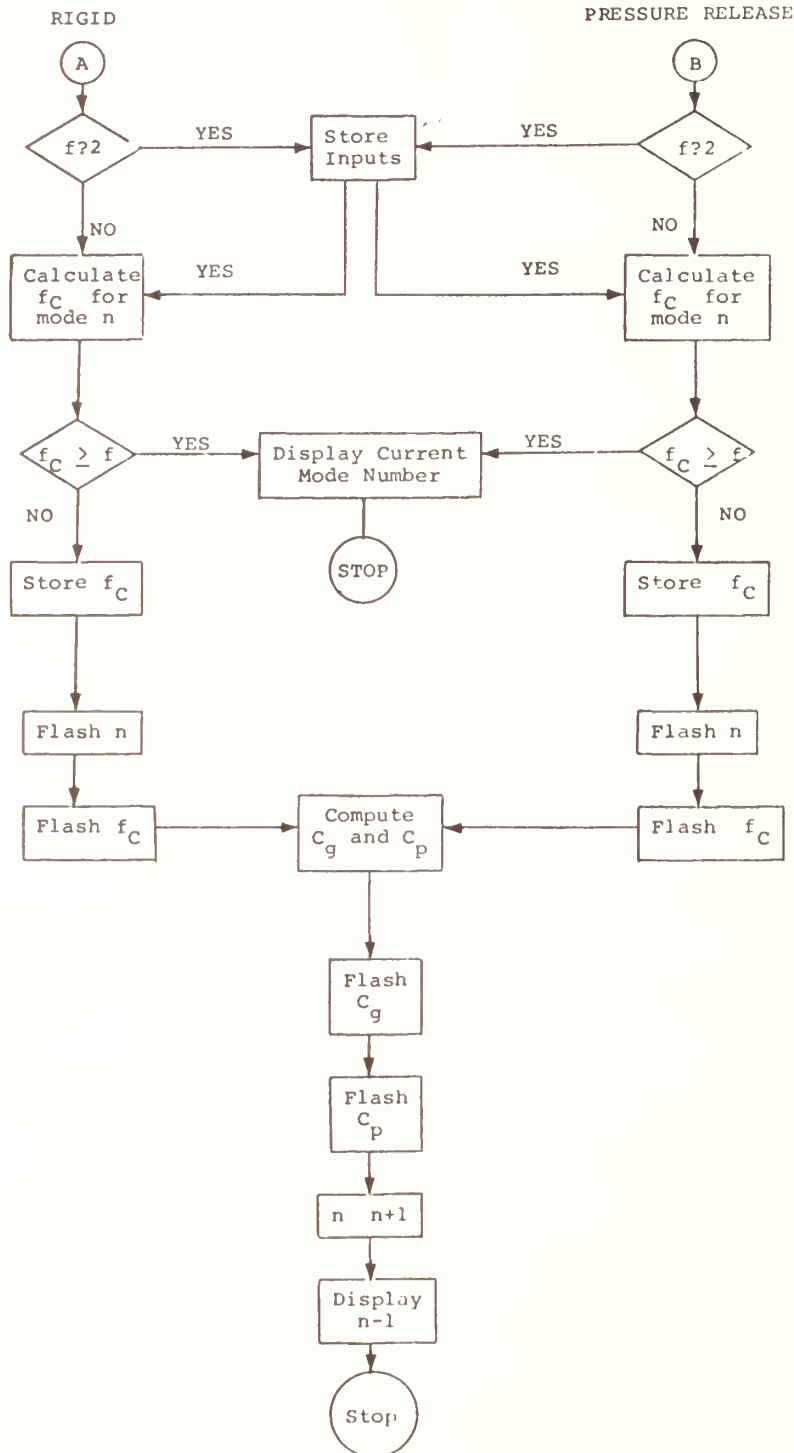


FIGURE 2. Group and Phase Velocity Geometry



Operator selects either rigid bottom or pressure release bottom



X. NORMAL MODE TRANSMISSION LOSSES by LT Michael D. Clary

A. Problem Statement

As an alternative to Ray Theory, the use of Normal Mode Theory provides a more exact approach to the solution of transmission loss problems. Given source frequency and depth, range to and depth of receiver, and modal values for the speed of sound and the absorption coefficient, the program user may either specify the effective pressure amplitude of the source at one meter and solve for modal effective pressure amplitudes and the resulting transmission losses, or if the modal pressures are input, the value of the one meter effective source pressure amplitude can be obtained.

B. Operational Analysis

Since this program is used primarily as a theoretical problem solver based on give data, no operational analysis is provided.

C. Computational Algorithm HP-67

1. Input source frequency, depth, receiver depth, effective pressure amplitude at 1 meter (source) or effective pressure amplitude of given model.
2. Input absorption coefficient for given mode, range between source and receiver, and speed of sound for the mode.
3. Compute sums required for transmission loss equation.
4. Compute and output coherent and/or incoherent transmission losses.

D. HP-67 Calculator Program

1. User Instructions

Step	Instruction	Input	Keys(s)	Output
1.	Load side one and two of program card			
2.	Program the eigenfunction for $z_n(z)$, $z_n(z_0)$ at LBL3 in the W/PGRM MODE. (Assume value of z and z_0 will be in the x-register upon initialization of each computation.) The last two program steps <u>must</u> be STO(i), h RTN. Return calculator to RUN MODE			
3.	Enter computational values			
a.	frequency of source in Hz	f	ENTER	f
b.	depth of source in meters	z_0	ENTER	z_0
c.	depth of receiver in meters	z	ENTER	z
d.	effective pressure amplitude at 1 m in μPa	$P(1)$	A	f
	or effective pressure amplitude of given value	$- P_n $	A	f
	enter as $- P_n $			
e.	absorption coefficient for the mode	α_n	ENTER	α_n
f.	range from source in meters	r	ENTER	
g.	speed of sound for the mode in meters/sec	C_n	B	$ P_n $ or $P(1)$
	Output will be either the mode value for the effective pressure amplitude or the value of the effective pressure amplitude at 1 m. All values are in μPa .			

Step	Instruction	Input	Key(s)	Output
4.	Sum the values of $P_n \sin(2\pi f_r)/C_n$ and $P_n \cos(2\pi f_r)/C_n$		C	n
5.	Sum the value of P_n^2		D	P_n^2
6.	Return to Step 2 for additional mode computations. Information for Steps 3a, b, c and d need not be re-entered. If all mode computations are completed, go to 7.			
7.	Compute transmission loss (in dB) for coherence for incoherence	f	E e	T.L. T.L.
8.	For new case go to Step 1			

2. Sample Problem

For a nearly isovelocity layer of water 50 m deep overlying a silt bottom rich in decaying organic matter, the following are found to be good approximate values for a source frequency of 50 Hz:

n	$Z_n(z)$	c_n	f_n (cutoff)	α_n
1	$0.2 \sin(0.050z)$	1550m/s	25 Hz	$1 \times 10^{-4}/m$
2	$0.2 \sin(0.105z)$	1630	33	2×10^{-4}
3	$0.2 \sin(0.189z)$	2110	45	4×10^{-4}
4	not excited	---	55	---

- a. Evaluate the effective amplitudes $|P_n|$ at a receiver at a range of 2 km and at a depth of 20 m. Determine both transmission loss values.

SOLUTION: (given that the source is at a depth of 50m)

1. Enter first eigenfunction $Z_n(z)$ under label 3.
Value of z (or z_0) will be in the X-register.
2. Enter f , z_0 , z and $P(1) = 10^7 \mu\text{Pa}$ for $n = 1$ and compute P_1 .
3. After recording value P_1 , sum the values for transmission loss calculations.
4. Repeat Steps 1-3 for all necessary values of n (three for the given example).
5. When all P_n 's have been obtained, calculate transmission loss, both coherent and incoherent cases.
6. Numerical answers:

$P_1 = 2.05 \times 10^4 \mu\text{Pa}$	$ P_1 = 2.05 \times 10^4 \mu\text{Pa}$
$P_2 = -2.54 \times 10^4 \mu\text{Pa}$	$ P_2 = 2.54 \times 10^4 \mu\text{Pa}$
$P_3 = 0.39 \times 10^3 \mu\text{Pa}$	$ P_3 = 0.39 \times 10^4 \mu\text{Pa}$

$$\text{TL}(\text{coherence}) = 53 \text{ dB}$$

$$\text{TL}(\text{incoherence}) = 50 \text{ dB}$$

Keystroke Sequence for Sample Problem a.

		GT03			2.-04 ENT;
092	*LBL3	21 03			2.+03 ENT;
093	.	-62			1630. GSES
094	0	00			2.54+04 ***
095	5	05			GSEC
096	x	-35			2.00+00 ***
097	SIN	41			GSED
098	.	-62			6.44+02 ***
099	2	02			GT03
100	x	-35			092 *LBL3 21 03
101	STO;	35 45			093 . -62
102	RTN	24			094 1 01
		50.00 ENT;			095 8 08
		50.00 ENT;			096 9 09
		20.00 ENT;			097 x -35
		1.+07 GSEA			098 SIN 41
		50.00 ***			099 . -62
		1.-04 ENT;			100 2 02
		2.+03 ENT;			101 x -35
		1550.00 GSEB			102 STO; 35 45
		2.05+04 ***			103 RTN 24
		GSEC			
		1.00+00 ***			4.-04 ENT;
		GSED			2.+03 ENT;
		4.22+08 ***			2110. GSEB
		GT03			3.92+02 ***
092	*LBL3	21 03			GSEC
093	.	-62			3.00+00 ***
094	1	01			GSEC
095	0	00			1.54+05 ***
096	5	05			GSEE
097	x	-35			53. ***
098	SIN	41			GSEB
099	.	-62			50. ***
100	2	02			
101	x	-35			
102	STO;	35 45			
103	RTN	24			

b. Using the same table values and the computed values for each P_n , verify that the value for $P(1)$ is in fact $10^7 \mu\text{Pa}$.

1. Enter first eigenfunction $Z_n(z)$ under label 3.
2. Enter computational values for $n = 1$ and compute $P(1)$.
Note that the values of P_n must be entered as the negative value of the absolute.
3. Calculate $P(1)$. Repeat Steps 1-3 for each value of n .
4. Numerical answers:

$$|P_1| = 2.05 \times 10^4 \mu\text{Pa} \quad P(1) = 9.98 \times 10^6 \approx 10^7 \mu\text{Pa}$$

$$|P_2| = 2.54 \times 10^4 \mu\text{Pa} \quad P(1) = 10^7 \mu\text{Pa}$$

$$|P_3| = 0.39 \times 10^3 \mu\text{Pa} \quad P(1) = 9.94 \times 10^6 \approx 10^7 \mu\text{Pa}$$

Keystroke Sequence for Sample Problem b.

```

GT03
50.00 ENT:
50.00 ENT:
20.00 ENT:
-2.05+04 GSEH
1.-04 ENT:
2.+03 ENT:
1550.00 GSEE
9.98+06 ***
GT03
50. ENT:
50. ENT:
20. ENT:
-2.54+04 GSEH
2.-04 ENT:
2.+03 ENT:
1630. GSEE
1.00+07 ***
GT03
50. ENT:
50. ENT:
20. ENT:
-0.39+03 GSEH
4.-04 ENT:
2.+03 ENT:
2110. GSEE
5.94+06 ***

```

3. Program Storage Allocation and Program Listing

Registers:

R0: f	S0: ΣP_n^2
R1: z_0	S1:
R2: z	S2:
R3: C_n	S3:
R4: α_n	S4: $\Sigma \sin(2\pi f_r)/C_n$
R5: r	S5:
R6: $P(1)$ or $- P_n $	S6: $\Sigma \cos(2\pi f_r)/C_n$
R7:	S7: $\Sigma \cos^2(2\pi f_r)/C_n$
R8:	S8: $\Sigma [\sin(2\pi f_r)/C_n] [\cos(2\pi f_r)/C_n]$
R9:	S9: n
RA: P_n or $P(1)$	
RB: $(2\pi f_r)/C_n$ and $\cos(2\pi f_r)/C_n$	
RC: $\sin(2\pi f_r)/C_n$	
RD: Coherent TL	
RE: Incoherent TL	
RI: Scratch	

Initial Flag Status and Use: OFF, Unused

Trig Mode: RAD

User Control Keys:

A: $f \uparrow z_0 \uparrow z \uparrow P(1) \text{ or } - P_n $	a:
B: $\alpha_n \uparrow r \uparrow C_n \rightarrow P_n \text{ or } P(1)$	b:
C: $\text{Sum sin, cos}(2\pi f_r)/C_n$	c:
D: $\text{Sum } P_n^2$	d:
E: $\rightarrow \text{Coherent TL}$	e: $\rightarrow \text{Incoherent TL}$

001	*LELA	21 11
002	RAD	16-22
003	CLRG	16-53
004	PZS	16-51
005	CLRG	16-53
006	ST06	35 06
007	R↓	-31
008	ST02	35 02
009	R↓	-31
010	ST01	35 01
011	R↓	-31
012	ST00	35 00
013	ETH	24
014	*LBL8	21 12
015	ST03	35 03
016	R↓	-31
017	ST05	35 05
018	R↓	-31
019	ST04	35 04
020	RCL6	36 06
021	X<0?	16-45
022	GT01	22 01
023	1	01
024	ST09	35 09
025	*LBL2	21 02
026	7	07
027	ST01	35 46
028	RCL1	36 01
029	GSE3	23 03
030	6	06
031	ST01	35 46
032	RCL2	36 02
033	GSE3	23 03
034	RCL5	36 05
035	X<0?	16-45
036	ETH	24
037	*LBL4	21 04
038	RCL7	36 07
039	RCL5	36 05
040	√X	54
041	÷	-24
042	RCL8	36 08
043	×	-35
044	RCL3	36 03
045	RCL0	36 00
046	÷	-24
047	√X	54
048	×	-35
049	RCL6	36 06
050	×	-35
051	RCL4	36 04
052	RCL5	36 05
053	×	-35
054	CHS	-22
055	e ^x	33
056	√	-35
057	ST0A	35 11

Enter values for f , Z_0 , Z , and either $P(1)$ or $-|P_n|$. Clears all registers, sets radian mode.

Enter values for α_n , r , and c_n

Determine calculations required.

1 is stored in R_9 if P_n is being calculated

Set up storage for $Z_n(Z_0)$ and $Z_n(z)$; branches to user-defined function to compute

Returns to Label 1 for calculations of $P(1)$

Compute P_n .

*Note that $|P_n|$ is stored in Register A, $|P_n|$ is displayed upon completion.

058	SCI	-12	
059	ABS	16 31	
060	RTN	24	
061	*LBL1	21 01	Stores in $ P_n $ in R_6 .
062	CHS	-22	-1 stores in R_9 to indicate that $P(1)$ is being calculated.
063	STO6	35 06	Calculate $Z_n(z_0)$, $Z_n(z)$
064	1	01	
065	CHS	-22	
066	STO9	35 09	
067	GSB2	23 02	
068	RCL5	36 05	
069	JX	54	
070	RCL6	36 06	
071	x	-35	
072	RCL3	36 03	
073	RCL0	36 00	
074	÷	-24	
075	JX	54	
076	÷	-24	
077	RCL7	36 07	
078	÷	-24	Compute $P(1)$
079	RCL8	36 08	
080	÷	-24	
081	RCL4	36 04	
082	RCL5	36 05	
083	x	-35	
084	CHS	-22	
085	e ^x	33	
086	÷	-24	
087	X<0?	16-45	
088	CHS	-22	
089	STOA	35 11	
090	SCI	-12	
091	RTN	24	
092	*LBL3	21 03	User-defined label to compute $Z_n(z_0)$ and $Z_n(z)$.
093	STO1	35 45	
094	RTN	24	
095	*LELC	21 13	
096	RCL5	36 05	
097	RCL0	36 00	
098	x	-35	
099	Pi	16-24	
100	2	02	
101	x	-35	
102	x	-35	
103	RCL3	36 03	Sums and stores the sine and cosine of
104	÷	-24	$\frac{2\pi f_r}{C_n}$
105	STOB	35 12	
106	SIN	41	
107	STOC	35 13	
108	RCLB	36 12	for use in calculations of T.L. (coherent)
109	COS	42	
110	STOB	35 12	
111	RCLA	36 11	
112	x	-35	
113	RCLC	36 13	
114	RCLA	36 11	

115	x	-35	
116	z+	56	
117	PTN	24	
118	*LELD	21 14	
119	RCLA	36 11	
120	X ²	53	
121	P ² S	16-51	Computes P_n^2
122	ST+0	35-55 00	
123	F ² S	16-51	
124	PTN	24	
125	*LBLE	21 15	
126	P ² S	16-51	Computes T.L. assuming coherence.
127	RCL4	36 04	
128	X ²	53	
129	RCL6	36 06	
130	X ²	53	
131	+	-55	
132	JX	54	
133	P ² S	16-51	
134	RCL6	36 06	
135	÷	-24	
136	LOG	16 32	
137	2	02	
138	0	00	
139	x	-35	
140	CHS	-22	
141	FIX	-11	
142	DSP0	-63 00	
143	STOD	35 14	
144	PTN	24	
145	*LBLE	21 16 15	Computes T.L. assuming incoherence.
146	P ² S	16-51	
147	RCL0	36 00	
148	P ² S	16-51	
149	JX	54	
150	RCL6	36 06	
151	÷	-24	
152	LOG	16 32	
153	2	02	
154	0	00	
155	x	-35	
156	CHS	-22	
157	FIX	-11	
158	DSP0	-63 00	
159	STOE	35 15	
160	RTN	24	
161	R/S	51	

E. Mathematical Analysis

The following four formulas are the basis for the mathematical computations (Ref. 1).

$$1. \quad P_n = P(1) \sqrt{\frac{C_n}{f}} \frac{z_n(z_0)}{\sqrt{r}} z_n(z) \exp(-\alpha_n r)$$

$$2. \quad P(1) = \frac{P_n \sqrt{r}}{\sqrt{(C_n/f)} z_n(z_0) z_n(z) \exp(-\alpha_n r)}$$

3. Coherent transmission loss

$$TL = -20 \log \left[\left[\left(\sum P_n \sin \frac{2f_r}{C_n} \right)^2 + \left(\sum P_n \cos \frac{2f_r}{C_n} \right)^2 \right]^{1/2} / P(1) \right]$$

4. Incoherent transmission loss

$$TL = -20 \log \left[(\sum P_n^2)^{1/2} / P(1) \right]$$

F. Reference

1. A. B. Coppens and J. V. Sanders, "An Introduction to the Sonar Equations with Applications," Technical Report NPS-61Sd76071, July 1976, Naval Postgraduate School, Monterey, CA 93940

XI. GOLDEN SECTION SEARCH by LT J. K. McDermott

A. Problem Statement

The minimum value of a unimodal function of one variable, $f(x)$, for a specified interval is determined by utilizing Golden Section Search Techniques, i.e.

$$\begin{array}{ll}\text{Minimize} & f(x) \\ \text{Subject to} & x \in I\end{array}$$

where $I = [a,b]$ is a closed interval in E_1 space.

B. Operational Analysis

Golden Section Search is a specific type of interval of uncertainty (IOU) method of single variable optimization which requires the selection of a specific interval. Once the interval has been selected, the program locates the value of x which will minimize a unimodal function $f(x)$ being evaluated within this specific interval. If a different interval is selected, a different x with a correspondingly different minimum functional value may be obtained.

Golden Section Search locates a local minimum and not the global minimum. A function which is not unimodal over the specific interval may produce an x value which does not provide the minimum (global) functional value within the interval. The behavior for non-unimodal functions is not predictable.

The functions which may be evaluated are limited to some degree by (i) the number of program steps available for user supplied function program listing (139-224), (ii) functions of one variable preferably unimodal over the IOU to avoid ambiguity, and (iii) user's programming capability and imagination.

Golden Section Search was originally suggested by J. Kiefer, "Sequential Minimax Search for a Maximum," Proc. Amer. Math. Soc., 4, no. 3, June 1953, pp. 502-506. The name traces back to Euclid's discovery that it is possible to divide any given line segment into two parts such that the ratio of the whole to the larger part equals the ratio of the larger part to the smaller. The division of a line in this manner came to be known as the Golden Section, both because it has several rather interesting geometric and numerical properties and because the proportions of the two parts seem pleasing to the eye.

The author (programmer) is indebted to Professor J. K. Hartman of the Naval Postgraduate School whose lectures and class notes form the bases of this HP-67/97 calculator program.

C. Computational Algorithm

Basic IOU Algorithm Structure (GSS)

1. Given initial IOU $I = [a, b]$ and function $f(x)$. Let K be a function evaluation, iteration counter.
2. Compute initial X_1 as

$$X_1 = a + \sigma(b-a) \quad \text{with} \quad \sigma = (3 - \sqrt{5})/2$$

Set $I_1 = I$ and $K = 2$.

3. At iteration K , interval I_{K-1} resulting from previous iteration contains best point (one producing smaller function value) thus far and its relative position is σ or $1-\sigma$. Place new point (X_K) symmetrically:

$$X_K = \text{ENDL} + \text{ENDR} - X_{\text{OLD}}$$

where $I_{K-1} = [\text{ENDL}, \text{ENDR}]$ and X_{OLD} provided the smaller function value between previous two evaluated points.

4. Compute $f(X_K)$.
5. Shorten IOU to $I_K \subset I_{K-1}$ with length $L_K < L_{K-1}$ from information $f(X_K)$ provides. Set $K = K+1$ and go to step 3 for the next iteration.

STOPPING RULE:

Stop when either $K = \text{NMAX}$ (present number of function iterations) or when $L_K \leq \text{RIOU}$ (preset required interval of uncertainty length).

HP-67 Computational Algorithm

1. Input user supplied function program listing in available program steps 139 through 224.
2. Input left endpoint of interval of uncertainty (ENDL).
3. Input right endpoint of interval of uncertainty (ENDR).
4. Input required length of the final interval of uncertainty (RIOU).
5. Input maximum number of function evaluations desired (NMAX)
6. Output final interval of uncertainty [ENDL,ENDR].
7. Output minimum (local) function value in interval.
8. Output X value that produces minimum function value.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1.	Enter program card			
2.	Select GTO f e		GTO f e	
3.	Slide W/PRGM-RUN switch to W/PRGM	User supplied function program		
4.	Slide W/PRGM-RUN switch to RUN			
5.	Enter left endpoint of interval of uncertainty	ENDL	A	ENDL
6.	Enter right endpoint of interval of uncertainty	ENDL	B	ENDR
7.	Enter required length of interval of uncertainty	RIOU	C	RIOU
8.	Enter maximum number of function evaluations to be performed	NMAX	D	NMAX
9.	Compute final interval of uncertainty, minimum function value, and corresponding X value	NONE	E	
	ENDL displayed when computation complete			ENDL
10.	Press R/S to display ENDR			ENDR
11.	Press R/S again to display f(X)			f(X)
12.	Press R/S once more to display X			X
13.	To repeat program press f CLREG and go to Step 5		f CL REG	

2. Sample Problem

a. minimize $f(x) = |x^2 - 16|$
 subject to $x \in I_0$

$$I_0 = [1, 16], \text{ RIOU} = 0.01, \text{ NMAX} = 25.$$

SOLUTION: $[3.99560, 4.00240]$ in 17 function evaluations

Function Value = 0.00162

Minimum Point x = 3.99980

User Supplied Program Listing

```

      GTOe
139  *LBLE 21 16 15
140  RCL E 36 15
141  X2 53
142  1 01
143  6 06
144  - -45
145  ABS 16 31
146  RTH 24
147  R/S 5:

```

Recalls x from Register E.

Function value is left in the
display (x-register).

```

      1.00 GSEA
      16.00 GSEB
      .01 GSEC
      25.00 GSED
      GSEE
3.99560 ***
      R/S
4.00240 ***
      R/S
0.00162 ***
      R/S
3.99980 ***

```

NOTE: First load provided program (with RUN position).
Select GTO f e. Move switch to W/PRGM. Enter user
supplied program for function. Move switch to run
position. Enter input and compute.

b. Minimize $f(x) = 20 - x + \frac{1}{(16-x)}$

Subject to $x \in I_0$

$I_0 = [10, 15.9]$, $RIOU = 0.01$, $NMAX = 25$

SOLUTION: $[14.99555, 15.00255]$ in 15 function evaluations

Function Value = 6.00000

Minimum Point $x = 14.99823$

User Supplied Program Listing

139	*LBLz	21 16 15	670e
140	2	02	
141	0	00	
142	PCLE	36 15	
143	-	-45	
144	1	01	
145	6	06	
146	PCLE	36 15	
147	-	-45	
148	1/X	52	
149	+	-55	
150	RTN	24	
151	R/S	51	

Recall x from Register E.

Result is left in x-register

10.00	656A
15.90	656B
.01	656C
25.00	656D
	656E
14.99555	***
	R/S
15.00255	***
	R/S
6.00000	***
	R/S
14.99623	***

NOTE: First load provided program with switch in RUN position. Select GTO f e. Move switch to W/PRGM. Enter user supplied program for function. Move switch to RUN position. Enter input and compute.

c. minimize $f(x) = \frac{x}{2} + \sin(\frac{\pi x}{2})$ (x in radians)

 subject to $x \in I_0$

$I_0 = [0,10]$, RIOU = 0.01, NMAX = 25.

SOLUTION: $[2.79094, 2.79827]$ in 16 function evaluations

Function value = 0.44890	{	local minimum (x = 0.0 is global)
Minimum point x = 2.79373		

User Supplied Program Listing

```
          GTOe
139  *LELe 21 16 15
140  RCLE  36 15
141      2      02
142  ÷      -24
143  ENT1   -21
144  ENT1   -21
145  P1     16-24
146  X      -35
147  SIN     41
148  +      -55
149  PTN     24
150  R/S     51
```

Recalls R5 for x utilization and
stores function value in R7
(GSB 1)

```
      0.00 GSEn
      10.00 GSEB
      .01 GSEC
      25.00 GSED
      GSEE
      2.79094 ***
           R/S
      2.79827 ***
           R/S
      0.44890 ***
           R/S
      2.79373 ***
```

NOTE: First load provided program with switch in RUN position
Select GTO f e. Move switch to W/PRGM. Enter user
supplied program for function. Move switch to RUN
position. Enter input and compute.

3. Program Storage Allocation

Registers:

R0: function counter	S0:	RA:
R1: ENDL	S1:	RB:
R2: ENDR	S2:	BC:
R3: RIOU	S3:	RD:
R4: NMAX	S4:	RE: Current X
R5: X_1	S5:	RI: 4 (decrement)
R6: X_2	S6:	
R7: F_1	S7:	
R8: F_2	S8:	
R9: not used	S9:	

NOTE: User supplied function can utilize sixteen registers.

Initial Flag Status and Use:


0: OFF, Unused	2: OFF, Unused
1: OFF, Unused	3: OFF, Unused

User Control Keys

A: Left endpoint (ENDL)	a:
B: Right endpoint (ENDR)	b:
C: Required interval of uncertainty (RIOU)	c:
D: Maximum function iterations (NMAX)	d:
E: Compute	e: User defined function

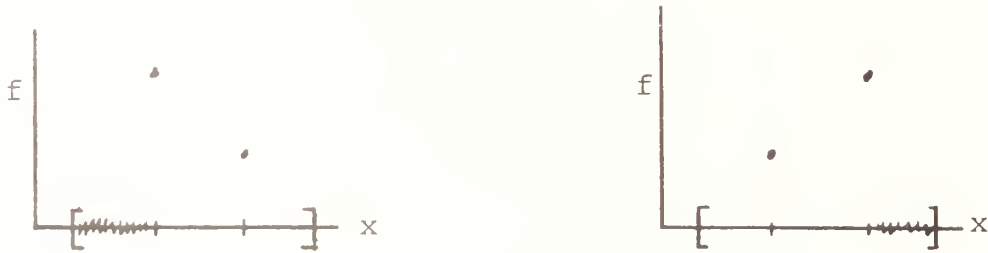
001	*LBLA	21 11	Input:
002	ST01	35 01	Left endpoint
003	RTN	24	Right endpoint
004	*LBLE	21 12	Required interval of uncertainty
005	ST02	35 02	Maximum number of function evaluations
006	RTN	24	
007	*LBLC	21 13	(ENDL, ENDR, RIOU, NMAX)
008	ST03	35 03	
009	RTN	24	
010	*LBLC	21 14	
011	ST04	35 04	
012	3	03	
013	ENT1	-21	Calculate Sigma
014	5	05	
015	JX	54	
016	-	-45	$\sigma = \frac{(3 - \sqrt{5})}{2}$
017	2	02	
018	=	-24	
019	RCL2	36 02	
020	RCL1	36 01	Calculate initial x_1
021	-	-45	
022	x	-35	
023	RCL1	36 01	$x_1 = ENDL + \sigma(ENDR - ENDL)$
024	+	-55	
025	ST05	35 05	
026	0	00	
027	ST06	35 06	Initialize counters
028	4	04	Set up display
029	ST01	35 45	Require radian calculations
030	RCL4	36 04	
031	DSF5	-63 05	
032	RAD	-16-22	
033	RTN	24	
034	*LBLE	21 15	
035	RCL5	36 05	
036	RCL1	36 01	
037	-	-45	Calculate initial x_2
038	RCL2	36 02	
039	XZY	-41	
040	-	-45	
041	ST06	35 06	
042	GSE1	23 01	Obtain initial function values
043	GSE8	23 08	(F_1 and F_2) from initial x_1 and x_2 .
044	GSE2	23 02	
045	GSE8	23 08	

046	*LBLA	21	16	11	Determine larger function value. If F_1
047	RCL8		36	08	larger go to branch two, otherwise go to
048	RCL7		36	07	branch one.
049	X>Y?		16	34	
050	GT03		22	03	
051	RCL6		36	06	X_2 becomes right endpoint of interval of
052	ST02		35	02	uncertainty
053	RCL1		36	01	
054	-		-45		Compare IOU < RIOU?
055	RCL3		36	03	If yes, print out final results
056	X≠Y		-41		
057	X≠Y?		16	35	
058	GT04		22	04	
059	RCL5		36	05	Old X_1 becomes X_2
060	ST06		35	06	Old F_1 becomes F_2
061	RCL7		36	07	
062	ST08		35	08	
063	BSB6		23	06	Determines new $X = (X_1)$ and store.
064	ST05		35	05	
065	RCL4		36	04	Has NMAX been exceeded? If yes, print error.
066	RCL0		36	00	
067	X>Y?		16	34	
068	GT00		22	00	
069	BSB1		23	01	Calculate F_1 and return to compare new
070	BSB8		23	08	values of F_1 and F_2 .
071	GT0A	22	16	11	
072	*LBL3		21	03	X_1 becomes left endpoint of interval
073	RCL2		36	02	of uncertainty
074	RCL5		36	05	
075	ST01		35	01	
076	-		-45		Compare IOU < RIOU?
077	RCL3		36	03	If yes, print out final results
078	X≠Y		-41		
079	X≠Y?		16	35	
080	GT04		22	04	
081	RCL6		36	06	Old X_2 becomes X_1
082	ST05		35	05	Old F_2 becomes F_1
083	RCL8		36	08	
084	ST07		35	07	
085	BSB6		23	06	Determines new $X = (X_2)$ and store.
086	ST06		35	06	
087	RCL4		36	04	Has NMAX been exceeded?
088	RCL0		36	00	If yes, print error.
089	X>Y?		16	34	
090	GT00		22	00	

091	GSE2	23 02	Calculate F_2 and return to compare new	
092	GSE8	23 08	values of F_1 and F_2	
093	GT0a	22 16 11	Increments function counter for determination of NMAX exceeded.	
094	*LBL8	21 02		
095	1	01		
096	ST+0	35-55 02		
097	RTN	24	Determines new X (X_1 or X_2 depending on which branch subroutine called from)	
098	*LBL6	21 05		
099	PCL2	36 02		
100	RCL5	36 05		
101	-	-45		
102	RCL1	36 01		
103	+	-55	Determines which function value with corresponding ENDL, ENDR, and X should be printed. (Smaller function value used.)	
104	RTN	24		
105	*LBL4	21 04		
106	PCL8	36 08		
107	RCL7	36 07		
108	XY9	16-34	Sets up stack for printout if F_1 small value.	
109	GT05	22 05		
110	RCL5	36 05		
111	RCL7	36 07		
112	PCL2	36 02	Sets up stack for printout if F_2 smaller value	
113	RCL1	36 01		
114	GT07	22 07		
115	*LBL5	21 05		
116	PCL6	36 06	Prints out final results.	
117	RCL8	36 08		
118	RCL2	36 02		
119	RCL1	36 01		
120	*LBL7	21 07		
121	R/S	51		
122	R4	-31	F ₁ Routine	
123	DEZ1	16 25 46		
124	GT07	22 07		
125	GT06	22 06		
126	PTN	24	F ₂ Routine	
127	*LBL1	21 01		
128	RCL5	36 05		
129	STOE	35 15		
130	GSEa	23 16 15	Use defined function	
131	ST07	35 07		
132	RTN	24		
133	*LBL2	21 02		
134	RCL6	36 06		
135	STOE	35 15		
136	GSEa	23 16 15		
137	ST08	35 08		
138	RTN	24		
139	*LBLc	21 16 15		
140	RTN	24		
141	R S	51		

E. Mathematical Analysis

For each X the function $f(X)$ is evaluated. By comparing two function values F_1 and F_2 the interval of uncertainty can be reduced as follows:



The placement of the X 's is determined by the Golden Section Search Technique utilizing the "Golden Ratio." Placement of the first X (X_1) determines the placement of all other X 's since all remaining points are placed symmetrically with respect to each point remaining in successive intervals of uncertainty.

In order to ensure that each IOU length is predictively independent of the function $f(x)$, X_1 and X_2 are placed symmetrically in the IOU. When the IOU is reduced, the new shorter IOU still contains the best point thus far achieved, so selecting a new X point will allow further reduction. (New X for each iteration.)

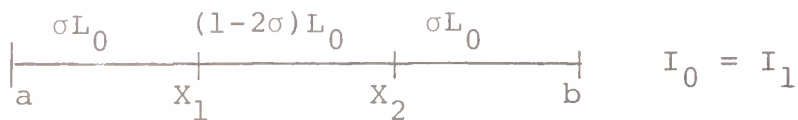
Golden Section Search selects X_1 to satisfy:

The relative position of the X points in the remaining IOU is the same at each iteration.

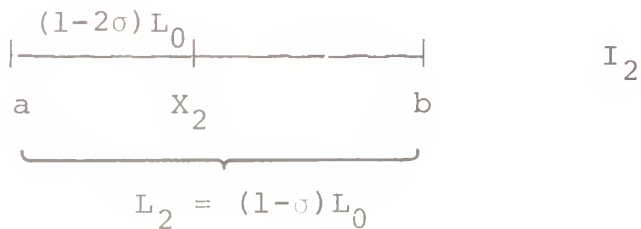
Explanation:



$I_1 = I_0$ since no reduction with one point. Relative position of X_1 in I_1 is $GL_0/L_0 = \sigma$. Now place X_2 symmetrically



Suppose $f(X_2) < f(X_1)$ and reduce IOU accordingly



Relative position of X_2 is

$$\frac{(1 - 2\sigma)L_0}{(1 - \sigma)L_0} = \frac{1 - 2\sigma}{1 - \sigma}$$

$$\sigma = \frac{1 - 2\sigma}{1 - \sigma} \Rightarrow \text{"relative position same at each iteration"}$$

$$\sigma^2 - 3\sigma + 1 = 0 \Rightarrow \sigma = \frac{3 \pm \sqrt{5}}{2} \quad \text{use root } \sigma = \frac{3 - \sqrt{5}}{2}$$

Choose $x_1 = \sigma b + (1-\sigma)a = a + \sigma(b-a)$.

Example: $n = 4$, $f(x) = (x - .303)^2$, $I_0 = [0, 1]$.

K	I_{K-1}	x_K	$f(x_K)$	I_K	L_K
0	--	--	--	$[0, 1]$	1
1	$[0, 1]$.382	$(.079)^2$	$[0, 1]$	1
2	$[0, 1]$.618	$(.315)^2$	$[0, .618]$	$.618 = (1-\sigma)$
3	$[0, .618]$.236	$(.067)^2$	$[0, .382]$	$.382 = (1-\sigma)^2$
4	$[0, .382]$.146	$(.157)^2$	$[\cdot146, .382]$	$.236 = (1-\sigma)^3$



FINAL ANSWER: x_1 (OPTIMAL) $e[.146, .382]$;

$x_3 = .236$ is best point evaluated thus far.

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